QCD sum rules for $\rho$-meson at finite density or temperature

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Informal group seminar

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contents based on:


YK, Sasaki & Weise [Phys. Rev. C81, 065203 (2010)]
Outline

- Introduction & motivation
- QCD sum rules for $\rho$-meson, in vacuum and in medium
- Finite energy sum rules at finite density
- Finite energy sum rules at finite temperature
- Summary and outlook
**Goal:** reliable framework of in-medium QCD sum rules for vector mesons ⇒ constraints for the in-medium spectral properties

**Motivation**

- Spontaneous chiral symmetry breaking:
  - quark condensate: $\langle \bar{q}q \rangle \neq 0$
  - Goldstone bosons: $\pi$, $K$, etc.
    - pion decay constant: $f_\pi \approx 92.4$ MeV
  - mass splitting of chiral partners
    (e.g. $\rho(770)-a_1(1250)$)

- Chiral symmetry restoration in nuclear medium:
  - degenerate chiral partners ⇒ modifications of hadron spectrum
Restoration scenarios in medium

- **Pole mass shift:**
  - masses of parity partners degenerate in medium.
  - moving toward each other or going to zero (Brown-Rho).


- **Width broadening:**
  - in-medium spectral functions are broadly distributed.
  - the continuum merges the broadened spectral distributions.

![Dropping Masses ?](image1)

![Melting Resonances ?](image2)
Dilepton spectroscopy

- Dilepton production in RHIC ($\gamma^* \rightarrow l^+ l^-$):
  - EM probe with pure information of the hot and/or dense region
  - Dilepton emission $\leftrightarrow$ in-medium vector-meson spectroscopy

<table>
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<th>$m_{ee}$ (GeV/c$^2$)</th>
<th>$dN/dM$ per 20 MeV</th>
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In-In NA60

- dropping mass
- broadening

Youngshin Kwon

QCD sum rules for $\rho$-meson at finite density or temperature
General review of QCD sum rules (in vacuum)
General review of QCD sum rules

- Current correlation function:
  \[ \Pi^{\mu\nu}(q) = i \int d^4 x \ e^{iq \cdot x} \langle T \ j^\mu(x) j^\nu(0) \rangle \]

  - isovector vector- and axialvector-currents:
    \[ j^\mu_P = \frac{1}{2} \left( \bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d \right), \quad j^\mu_A = \frac{1}{2} \left( \bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d \right) \]

  - invariant correlator: \( \Pi(q^2) = \frac{1}{3} g_{\mu\nu} \Pi^{\mu\nu}(q) \)

- Operator product expansion (quark & gluon d.o.f.) at large \( Q^2 = -q^2 \):
  \[ \frac{12\pi^2}{Q^2} \Pi(Q^2) = -c_0 \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \cdots \]

- Spectral representation (hadronic d.o.f.) at resonance region:
  \[ \Pi(q^2) = \Pi(0) + q^2 \Pi'(0) + \frac{q^4}{4\pi} \int ds \frac{\text{Im} \Pi(s)}{s^2(s - q^2 - i\epsilon)} \]
General review of QCD sum rules

- Borel transformation:

\[
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
\]

- Dimensionless spectral function: \( R(s) \equiv -\frac{12\pi}{s} \text{Im} \, \Pi(s) \)

- Coefficients \( c_n \):

\[
c_0 = \frac{3}{2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \cdots, \quad c_1 \propto m_q^2: \text{negligibly small}
\]

\[
\frac{\pi^2}{2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \pm 6\pi^2 \left( m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle \right)
\]

- \( 0.005 \pm 0.004 \text{ GeV}^4 \)

- \( \approx -m_\pi^2 f_\pi^2 = -(0.11 \text{ GeV})^4 \)

Ioffe [ PPNP 56, 232 (2006) ]

[ Gellman-Oaks-Renner ]
General review of QCD sum rules

Borel transformation:

\[
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
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\[
c_2 = \frac{\pi^2}{2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \pm 6\pi^2 \left( m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle \right)
\]

\[
c_3 \propto \mp \langle (\bar{q}q)^2 \rangle \text{ uncertain value}
\]
General review of QCD sum rules

◎ Borel transformation:

\[
12\pi^2 \Pi(0) + \int_0^\infty ds \, R(s) \, e^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots
\]

- dimensionless spectral function: \( R(s) \equiv -\frac{12\pi}{s} \text{Im} \Pi(s) \)
- expand for \( s_0 \ll M^2 \) and compare term by term

◎ Coefficients \( c_n \):

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c_0 = \frac{3}{2} \left(1 + \frac{\alpha_s}{\pi}\right) + \cdots, \quad \quad c_1 \propto m_q^2 : \text{negligibly small}
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c_3 \propto \mp \left\langle (\bar{q}q)^2 \right\rangle \text{uncertain value}
\]
Hierarchical sum rules for moments of $R(s)$:

- **0th moment**: 
  \[ \int_0^{s_0} ds R(s) = s_0 c_0 + c_1 - 12\pi^2 \Pi(0) \]

- **1st moment**: 
  \[ \int_0^{s_0} ds s R(s) = \frac{s_0^2}{2} c_0 - c_2 \]

Spectral distribution (resonance + continuum):

\[ R(s) = R_\rho(s) \theta(s_0 - s) + R_c(s) \theta(s - s_0) \]

Assumption for vector channel:

\[ \sqrt{s_0} \approx 4\pi f_\pi \]
**Finite energy sum rules**

- Hierarchy of finite energy sum rules for moments of $R(s)$:
  
  \[0^{th}\text{ moment} : \int_0^{s_0} ds \, R(s) = s_0 c_0 + c_1 - 12\pi^2 \Pi(0)\]
  
  \[1^{st}\text{ moment} : \int_0^{s_0} ds \, s \, R(s) = \frac{s_0^2}{2} c_0 - c_2\]

- Spectral distribution (resonance + continuum):
  
  \[R(s) = R_\rho(s) \theta(s_0 - s) + c_0 \theta(s - s_0)\]

  - Assumption for vector channel;
    
    \[\sqrt{s_0} \approx 4\pi f_\pi\]
Consistency with current algebra

- Identification of $\sqrt{s_0}$ with $\Lambda_{\text{CSB}} \approx 4\pi f_\pi$:

  $\bar{K}^* \bar{K} \rho \pi a_1^0$

  Mass [GeV]

  - Goldstone Boson
  - Dipole
  - Axial Dipole

  $\pi^-$

  $\rho^-$

  $a_1^+$

  Gap $4\pi f_\pi$

- KSRF relation
  - Riazuddin & Fayyazuddin [ PR 147, 1071 (1966) ]

- Weinberg sum rules
  - Weinberg [ PRL 18, 507 (1967) ]
  \[ m_{a_1} = \sqrt{2} m_\rho = 4\pi f_\pi \]
Consistency with current algebra

- Identification of \( \sqrt{s_0} \) with \( \Lambda_{\text{CSB}} \approx 4\pi f_\pi \):

**KSRF relation**

Kawarabayashi & Suzuki [ PRL 16, 255 (1966) ]
Riazuddin & Fayyazuddin [ PR 147, 1071 (1966) ]

**Weinberg sum rules**

Weinberg [ PRL 18, 507 (1967) ]

\[ m_{a1} = \sqrt{2} m_\rho = 4\pi f_\pi \]

0th moment:

\[ \int_0^{s_0} ds R_\rho(s) = \frac{3}{2} s_0 \]

\[ \Rightarrow m_\rho^2 = 2g^2 f_\pi^2 \]

1st moment:

\[ \int_0^{s_0} ds sR_\rho(s) = \frac{3}{4} s_0^2 \]

\[ \Rightarrow g = 2\pi \]
Vacuum sum rule analysis

- **Input:** $R_{\rho}(s)$ from chiral effective field theory + vector mesons (VMD)

$$\sqrt{s_0} = 1.14 \pm 0.01 \text{ GeV} \approx 4\pi f_\pi$$

$$\tilde{m}_\rho = \sqrt{\frac{\int ds \ s R(s)}{\int ds \ R(s)}} = 0.78 \pm 0.01 \text{ GeV}$$
Vacuum sum rule analysis

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\[ \bar{m}_\rho \equiv \sqrt{\frac{\int ds \, s R(s)}{\int ds R(s)}} = 0.78 \pm 0.01 \text{ GeV} \]
replace the Heaviside step function with a ramp function:

\[ R(s) = R_\rho(s) \theta(s_2 - s) + R_c(s) W(s) \]

with the weight function \( W(s) \)

\[
W(x) = \begin{cases} 
0 & \text{for } x \leq s_1 \\
\frac{x - s_1}{s_2 - s_1} & \text{for } s_1 \leq x \leq s_2 \\
1 & \text{for } x \geq s_2 
\end{cases}
\]

No dependence on details of the threshold modeling
Sensitivity to threshold modeling

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1 & \text{for } x \geq s_2
\end{cases}
\]

- No dependence on details of the threshold modeling

\[ s_0 = (s_1 + s_2)/2 \]
Finite energy sum rules at finite density
Medium-modification of the sum rules

- The existence of nuclear matter causes breaking Lorentz invariance:
  - two invariant correlator: longitudinal and transverse parts.
  - choosing a preferred reference frame of the medium ($q = 0$), longitudinal and transverse correlators coincide.

\[ \Pi_L(\omega, q = 0) = \Pi_T(\omega, q = 0) \equiv \Pi(\omega, q = 0) \]

- New operators with spin appear due to the broken Lorentz invariance:
  - the first moment of in-medium FESR involves twist-2 operator (e.g. $\langle \bar{q} \gamma_\nu D_\mu q \rangle$) to be considered.

- Medium-dependence in the OPE side contributes only to the condensates:
  - non-perturbative contributions in OPE appear to be clearly separated into the condensates.
  - medium effects are non-perturbative.
Density-dependence of OPE


- Expectation value: vacuum \( \rightarrow \) ground state of nuclear matter

\[ \langle 0 | O | 0 \rangle \equiv \langle O \rangle_0 \rightarrow \langle O \rangle_{\rho_N} = \langle N | O | N \rangle \]

- In-medium coefficients: \( c_n \rightarrow c_n + \delta c_n \)

\[ \delta c_2 = -3\pi^2 \left( \frac{4}{27} M_N^{(0)} - 2\sigma_N - A_1 M_N \right) \rho_N \]

- Density dependence of gluon condensate
  \( (M_N^{(0)} \approx 0.88 \text{ GeV}) \)

- Density dependence of quark condensate
  \( (\sigma_N \approx 45 \text{ MeV}) \)

- First moment of parton distribution from DIS
  \( (A_1 \approx 1.24) \)
Spectral functions at finite density

- $\rho$-meson spectral functions in nuclear medium ($\rho_N = \rho_0 = 0.17\text{ fm}^{-3}$):

- KKW: SU(3) chiral dynamics with vector meson dominance

- RW: particle-hole excitations ($\Delta(1232)-h$ and $N^*(1520)-h$)
Results for $\rho$-meson at finite density

In vacuum: $\sqrt{s_0} \approx 1.14 \text{ GeV} \approx 4\pi f_\pi$

$$\tilde{m}^2 \equiv \frac{\int_0^{s_0} ds \ s R(s)}{\int_0^{s_0} ds \ R(s)}$$

In-medium KKW spectrum:
$$\sqrt{s_0^*} \approx 1.00 \pm 0.02 \text{ GeV}$$
$$\sqrt{\frac{s_0^*}{s_0}} = \frac{f_{\pi}^*}{f_\pi} \approx 0.87 \approx \frac{\tilde{m}^*}{\tilde{m}} : \text{BR-scaling}$$

In-medium RW spectrum:
$$\sqrt{s_0^*} \approx 1.09 \pm 0.01 \text{ GeV}$$
$$\sqrt{\frac{s_0^*}{s_0}} \approx \frac{\tilde{m}^*}{\tilde{m}} \approx 0.96$$

Kwon, Procura & Weise [ PRC 78, 055203 (2008) ]
Finite energy sum rules at finite temperature
Temperature-dependence of OPE


○ Thermal expectation value:

\[
\langle O \rangle_0 \rightarrow \langle O \rangle_T = \frac{\text{Tr} O \exp(-H/T)}{\text{Tr} \exp(-H/T)}
\]

○ In-medium coefficients: \( c_n \rightarrow c_n + \delta c_n \)

\[
\delta c_2 = -\frac{3}{2} \left( \frac{2}{9} \right) = 3 + A_1 \left( m_{\pi}^2 T^2 \right) \int_{m_{\pi}/T}^{\infty} dy \frac{\sqrt{y^2 - (m_{\pi} T)^2}}{e^y - 1}
\]

- \( T \)-dependence of gluon condensate
- \( T \)-dependence of quark condensate
- first moment of parton distribution from DIS
Mixing of vector and axialvector:

\[
R_V(s, T) = R_V(s, 0) (1 - \epsilon(T)) + R_A(s, 0) \epsilon(T)
\]

\[
R_A(s, T) = R_A(s, 0) (1 - \epsilon(T)) + R_V(s, 0) \epsilon(T)
\]


- the mixing parameter \(\epsilon(T)\) is given by the thermal pion loop:

\[
\epsilon(T) = \frac{2}{f^2} \int \frac{d^3k}{\omega (2k)^3} \frac{1}{e^{\omega/T} - 1} \rightarrow \frac{T^2}{6 f^2} \quad \text{as } m_\pi \rightarrow 0
\]

where \(\omega^2 = k^2 + m_\pi^2\).

- At critical temperature where \(\epsilon \approx \frac{1}{2}\), \(R_V\) and \(R_A\) become identical.
Mixing of finite-width spectrum

- Spectral functions with finite decay width:

\[ R_{\rho}(s,T) \]

\[ R_{a_1}(s,T) \]
Sum rule result for vector channel:

- Average $\rho$-meson mass:
  \[
  \bar{m}_\rho^2 = \frac{\int_{s_0}^s ds \ s R_\rho(s)}{\int_{s_0}^s ds \ R_\rho(s)}
  \]

- Comparison with ChPT:
  \[
  f_\pi(T) = f_\pi \left( 1 - \frac{1}{2} \epsilon(T) \right)
  \]
Simple test beyond $V$-$A$ mixing

- Dropping pole mass in addition to the $V$-$A$ mixing:

\[
\begin{align*}
R_{\rho}(s, 0) &= F_{\rho}^2 \delta \left( s - m_{\rho}^2 \right) \\
R_{a}(s, 0) &= F_{a}^2 \delta \left( s - m_{a}^2 \right) \\
R_{\rho}(s, T) &= R_{\rho}(s, 0) \left( 1 - \epsilon \right) + R_{a}(s, 0) \epsilon 
\end{align*}
\]

Brown-Rho scaling hypothesis:

\[
m_{\rho}^2 \to m_{\rho}^2 \left( 1 - \frac{1}{2} \epsilon(T) \right)^2
\]

$\Rightarrow$ better agreement: \( \sqrt{s_0} = 4\pi f_{\pi}(T) \)
About four-quark condensates

- Sum rules for 0th and 1st moments: RHS quantities are accurately determined (pQCD and leading condensates)
- Sum rules for 2nd moment: involving four-quark condensates

\[
\int_0^{s_0} ds \, s^2 R(s) = \frac{s_0^3}{3} + c_3
\]

\[
c_3 = -6\pi^3 \alpha_s \left[ \langle (\bar{u} \gamma_\mu \gamma_5 \lambda^a u - \bar{d} \gamma_\mu \gamma_5 \lambda^a d)^2 \rangle + \frac{2}{9} \langle (\bar{u} \gamma_\mu \lambda^a u + \bar{d} \gamma_\mu \lambda^a d) \sum_{q=u,d,s} \bar{q} \gamma^\mu \lambda^a q \rangle \right]
\]

- Ground state saturation \((\kappa = 1)\)

\[
\langle (\bar{q} \gamma_\mu \gamma_5 \lambda^a q)^2 \rangle = -\langle (\bar{q} \gamma_\mu \lambda^a q)^2 \rangle = \frac{16}{9} \kappa \langle \bar{q} \gamma_5 q \rangle^2
\]

valid approximation? \(\Rightarrow\) Always \(\kappa > 3\) and large uncertainties.
The sum rules for the lowest two moments of the \( \rho \)-meson spectral function involve perturbative contributions and only leading condensates as small corrections: accuracy both in vacuum and in medium

Chiral gap scale: \( 4\pi f_\pi \) meaningful both in vacuum and in-medium.

For broad spectral distributions, “mass shift” vs. “broadening” discussion must be specified in terms of first moment.

Brown-Rho scaling as a statement involving the lowest two moments in the window of low-mass enhancement.

Further step: extension to nonvanishing three-momentum.
Thank you for your attention!