Neutron star and low energy QCD


Internal seminar

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Motivation – astrophysical observation

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- Tidal deformability estimated from GW170817 (LIGO-Virgo)
Outline

I. Massive neutron stars and quasi-hadron picture
   • Quantum hadron dynamics and mean-field phenomenology
   • Low energy QCD symmetry breaking and sum rules for quasi-baryon states

II. Gravitational wave observation and quarkyonic matter concept
   • Large $N_c$ QCD in dense medium
   • Excluded-volume model

III. Three-flavor extension of excluded-volume model
Massive states

• More observation of $2M_\odot$ states from a compact binary

(Science 340 (2013) 1233232 J. Antoniadis et al.)

• To reach such a massive state, hard enough EOS is required

  ▪ Tolman-Oppenheimer-Volkof equation

$$\frac{dp(r)}{dr} = \frac{G(\epsilon(r) + p(r))(M(r) + 4\pi r^3 p(r))}{r(r - 2GM(r))}, \quad \frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r), \quad p = -\epsilon + \mu_B n_B$$

  ▪ Intrinsic strong repulsive interaction is required

(Nat.Astron.4 (2019) 72 H. T. Cromartie et al.)
Nuclear symmetry energy

- For continuous (infinite) matter

Similarly, from equation of state

\[
\frac{E(\rho_N, I)}{A} = E_0(\rho_N) + E_{\text{sym}}(\rho_N) I^2 + O(I^4) + \cdots
\]

\[
E_{\text{sym}}(\rho_N) = \frac{1}{2!} \frac{\partial^2}{\partial I^2} \tilde{E}(\rho_N, I) \quad I = (\rho_n - \rho_p)/\rho_N
\]

If one assume linear density dependent potential, the symmetry energy can be easily read off from potential

\[
E_{\text{sym}} = \frac{1}{4I} (E_n(\rho, I) - E_p(\rho, I)) \quad \text{dependent on } (\rho, I)
\]

- Quasi-particles?

In continuous matter, nuclear potential can be understood as self-energy of quasi-nucleon

Energy dispersion relation can be written with self-energies (Relativistic mean-field theory)

\[
G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | T \{ \bar{\psi}(x) \psi(0) \} | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{\Sigma_v}}{(q_0 - E_q)(q_0 - E_0)} \quad \text{(Near quasi-pole)}
\]
Nuclear symmetry energy – stiff or soft?

- Nuclear phenomenology (RMFT)  
  (Phys. Rept. 410 (2005) 335 V. Baran et al.)

\[ NLp\delta \text{ model (Iso-spin dependent interaction)} \]

\[
\mathcal{L} = i \gamma_{\mu} \partial^{\mu} - (M_N - g_\sigma \phi - g_\delta \vec{\tau} \cdot \vec{\delta}) - g_\omega \gamma_{\mu} \omega^{\mu} \\
- g_\rho \gamma^{\mu} \vec{\tau} \cdot \vec{b}_\mu \psi + \frac{1}{2} (\partial_{\mu} \phi \partial^{\mu} \phi - m_\sigma^2 \phi^2) - U(\phi) \\
+ \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} + \frac{1}{2} m_\rho^2 \vec{b}_\mu \cdot \vec{b}^{\mu} + \frac{1}{2} (\partial_{\mu} \vec{\delta} \cdot \partial^{\mu} \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}.
\]

\[
E_{\text{sym}} = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{m^*}{E_F^*} \right)^2 \right] \rho_B \\
\approx \frac{1}{6} \frac{k_F^2}{E_F^*} + \Sigma_0 + \frac{m^*}{E_F^*} \Sigma_\delta
\]

Iso-spin scalar channel (attraction) becomes weaker at dense matter ⇒ stiffly increases

If symmetry energy is stiff, neutron sea will become unstable at dense regime
- leads to \( nn \rightarrow p\Delta \)- type scattering
- iso-spin density becomes lower
- subsequently changed hadron yield such as \( \pi^-/\pi^+ \) from final state will become smaller
Hyperons and EOS

• Hyperons (S≠0) in medium (in vacuum, \(M_N \sim 940\) MeV, \(M_\Lambda \sim 1115\) MeV, \(M_\Sigma \sim 1190\) MeV)
  
  \(\Lambda\) is bounded (V \(\sim -30\) MeV)  
  \(\Sigma\) potential is repulsive (V \(\sim +100\) MeV)

\[
-U(r) = 56.67f(r) - 30.21f^2(r)
\]

\[
U(r) = (V_0 + iW_0)f(r) + V_{\text{spin}}(r, \vec{r}, \vec{\sigma}) + V_{\text{Coulomb}}(r)
\]

<table>
<thead>
<tr>
<th>(U_{\Sigma}^a)</th>
<th>(U_{\Sigma}^{sa})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_0) (MeV)</td>
<td>(+150) (-10)</td>
</tr>
<tr>
<td>(W_0) (MeV)</td>
<td>(-15) (-10)</td>
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<tr>
<td>(V_{SO}) (MeV)</td>
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<tr>
<td>(c) (fm)</td>
<td>(3.3^c)</td>
</tr>
<tr>
<td>(z) (fm)</td>
<td>0.67</td>
</tr>
</tbody>
</table>

(PRC38 (1988) 2700 D. J. Millener et. al.)

(PRL89 (2002) 072301 H. Noumi et al.)

• If hyperon energy is a similar order of nucleon energy? (\(\rho > \rho_0, \ I=1\))

New degree of freedom (hyperon) can appear in the nuclear matter
  → matter becomes softer → maximum neutron star mass will be bounded around \(1.5M_\odot\)
  \(2M_\odot\) neutron star has been observed (Nature 467 (2010) 1081 P. B. Demorest et al.)

→ should we exclude hyperons in neutron star? How to obtain such a stiff EOS?
  → related to density evolution of hyperon energy and symmetry energy
QCD symmetry breaking

• Classical symmetry

\[ \mathcal{L}_{\text{E}}^{\text{QCD}} = \frac{1}{2} \text{Tr}[F_{\mu\nu} F_{\mu\nu}] + \sum_i \bar{q}_i D_i q_i + \mathcal{L}_{\text{ghost}} + \mathcal{L}_{\text{g.f.}} + \cdots \quad \text{U}(1) \times SU(2)_L \times SU(2)_R \times SU(3) \]

• After quantum correction

\[ \mathcal{L}_{\text{E}}^{\text{QCD}} = \frac{1}{2} \text{Tr}[F_{\mu\nu} F_{\mu\nu}] + \sum_i \bar{q}_i D_i q_i + \mathcal{L}_{\text{ghost}} + \mathcal{L}_{\text{g.f.}} + \frac{ig^2}{8\pi^2} \text{Tr}[F_{\mu\nu} \tilde{F}_{\mu\nu}] \theta + \cdots \]

\[ \partial^\mu j_5^\mu = \frac{g^2 n_f}{8\pi^2} \text{Tr}[F_{\mu\nu} \tilde{F}_{\mu\nu}] \equiv \nu = n_L - n_R \quad (F_{\mu\nu} \equiv \frac{\epsilon_{\mu\nu\alpha\beta}}{2!} F^{\alpha\beta}) \]

Axial charge is not conserved

For \( \nu=1 \) configuration, classical solution (instanton) can be found as

\[ A_{\text{inst.}}^{\mu}(x) = \frac{2}{g} \frac{\eta_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2 + \lambda^2}, \quad \text{O}(4) \simeq SU(2) \times SU(2) \rightarrow \text{trapped color already breaks symmetry} \]

which resides in closed Wilson loop \( \exp \int_C dx^\mu A_\mu(x), \quad F^{\mu\nu} = 0 \) (zero curvature on the contour) (Belavin, Polyakov, Schwartz, Tyupkin, PLB59(1975)85)
QCD symmetry breaking

- Fermionic zero-modes in $\nu > 0$ configuration

\[ i\mathcal{P}_{\text{inst.}} q = 0, \quad (i\mathcal{P}_{\text{inst.}})^2 q = 0 \]

\[ (i\mathcal{P}_{\text{inst.}})^2 q = 0 \quad \text{leads to} \quad \left( -D_{\text{inst.}}^2 + \frac{1}{2} \sigma_{\mu\nu} F_{\mu\nu}^{\text{inst.}} \right) q_L = 0 \quad \text{and} \quad -D_{\text{inst.}}^2 q_R = 0 \quad (-D_{\text{inst.}}^2 > 0) \]

different topological configuration measures different axial charge

- $\theta$ vacuum

\[ Z_\theta = \sum_\nu \exp \left[ - \int d^4 x E \mathcal{L}_{E,\nu}^{\text{QCD}} \right] \rightarrow \text{all possible topological configuration contributes} \]

helicity bases can be correlated via Instantons

For a given gauge configuration $\langle \bar{q}q \rangle \neq 0$ breaks chiral symmetry

$U_V(1) \times U_A(1) \times SU(2)_V \times SU(2)_A \times SU(3)$

hadron property can be described via the symmetry breaking pattern
QCD sum rules

- Interpolating current correlator
  \[ \Pi(q) = i \int d^4x e^{iqx} \langle \Psi_0 | T[\bar{\eta}(x)\eta(0)] | \Psi_0 \rangle \]
  - Quasi-particle state will be extracted from the overlap
  - We need to construct proper interpolating current which can be strongly overlapped with object hadron state n, p, \Lambda, \Sigma, and \Delta resonances

- Operator product expansion (Example: 2-quark condensate diagram)
  \[
  \Pi_i(q^2) = \sum_n C_n^i(q^2, \mu) \langle \hat{O}_n(\mu) \rangle_0
  \]
  - Vacuum expectation value of localized operators
  - Short distance \( q > \mu \)
  - Long distance \( q < \mu \)

Overlap
Between \( \bar{\eta}(0) \) and \( \Psi_N^+ | 0 \rangle \)
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II. Gravitational wave observation and quarkyonic matter concept
   • Large $N_c$ QCD in dense medium
   • Single flavor excluded-volume model

III. Three-flavor extension of excluded-volume model
Gravitational wave

- Tidal deformability observed from GW170817 (LIGO-Virgo)

**Inspiral of two objects** ($R_1 > 0$)

$$ \Lambda = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4 \Lambda_1 + (m_2 + 12m_1)m_2^4 \Lambda_2}{(m_1 + m_2)^5} $$

**Successive analysis** (PRL121.161101 B.P. Abbott et al.)

$$ \tilde{\Lambda}(1.4M_\odot) < 580, \quad P(2n_0) = 3.5^{+2.7}_{-1.7} \times 10^{34}\text{dyn/cm}^2, \quad P(6n_0) = 9.0^{+7.9}_{-2.6} \times 10^{34}\text{dyn/cm}^2, \quad R_{1.4} = 11.9^{+1.4}_{-1.4} \text{ km} $$
Possible equation of state

- Example based on machinery computations

By polytropes interpolation (Quark Matter 2018, NPA982.36 A. Vuorinen)

- Gradual “soft” increment and “stiff” increment (small $v_s^2$ after sudden increment)
- The 2$^{nd}$ soft part is constrained by pQCD limit
Speed of sound

- Fast and slow speed

\[
v_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{n}{\mu (\partial n/\partial \mu)}, \quad \frac{\delta \mu}{\mu} \sim v_s^2 \frac{\delta n}{n}, \quad v_s^2 < 1 \text{ (causality)}
\]

- If \(v_s^2 \sim 1\), the unit increment of log scale will be directly proportional to each other → requires tremendously large repulsive interaction

- At some points, this huge force should be turned off to satisfy the \(\bar{\Lambda} < 800\) constraint
Large $N_c$ QCD and quarkyonic matter

- $g_T^2N_c = \text{const.}, N_c \to \infty$ (Nucl.Phys.B72 (1974) 461 G. 't Hooft)

- Effectively $U(N_c \to \infty)$ gauge theory, anomaly free
  \[ \partial^\mu j_\mu^5 = \frac{g^2n_f}{8\pi^2N_c} \text{Tr}[F^{\mu\nu}\tilde{F}_{\mu\nu}] \to 0 \]

\[ i \quad \begin{array}{ccc}
  \mu & \quad \delta_{\mu\nu} & \quad \text{(vector field)} \\
  j & \frac{k^2 - ie}{k^2 - ie} & \\
  v & \quad \text{(F.P. ghost)} \\
  j & \quad \frac{1}{k^2 - ie} & \\
  \end{array} \]

\[ i \quad \begin{array}{ccc}
  \frac{1}{m(q) + i\gamma k - ie} & \quad \text{(quark } q^q_i) \\
  a & \quad \text{(Fermi statistics)} \\
  \end{array} \]

\[ ig \{ \delta_{\alpha\nu}(k-q)_\mu + \delta_{\alpha\mu}(p-k)_\nu + \delta_{\mu\nu}(q-p)_{\alpha} \} \]

\[ g^2 \{ 2\delta_{\alpha\mu}\delta_{\nu\beta} - \delta_{\alpha\beta}\delta_{\mu\nu} - \delta_{\mu\nu}\delta_{\alpha\beta} \} \]
Large $N_c$ QCD and quarkyonic matter

- Faces ($F = L + I$), internal lines ($P$), vertices ($V$), quark loops ($L$), index loops ($I$)
- Coupling factor

\[
\begin{aligned}
  r &= g^{V_3 + 2V_4} N^I \\
  &= g^{2P - 2V} N^{F - L} \\
(2P &= \sum_n n V_n, \quad F - P + V = 2 - 2H) \\
  &= (g^{2N})^{1/3} V_3 + V_4 N^{2 - 2H - L}
\end{aligned}
\]

![Diagram showing the relationship between $1/N_c$ and $(1/N_c)^2$.](a) $1/N_c$, (b) $(1/N_c)^2$
Large $N_c$ QCD and quarkyonic matter

- $g_T^2 N_c = \text{const.}, \ N_c \to \infty$ (Nucl.Phys.B160 (1974) 57 E. Witten)

  - Easier counting

  \[
  \frac{g}{\sqrt{N_c}} \to \begin{array}{c}
  i \\
  j \\
  k \\
  \end{array} \\
  \begin{array}{c}
  i \\
  j \\
  k \\
  \end{array} \\
  \sim \frac{1}{N_c} \left( N_c \right)^2 \sim 1/N_c
  
  - Planar diagram dominates

  \begin{align*}
  \sim & (1/N_c)^6 (N_c^3)^3 \sim (1/N_c)^3 \\
  \sim & (1/N_c)^6 (N_c)^1 \sim (1/N_c)^5
  \end{align*}
Large $N_c$ QCD and quarkyonic matter

- $g_T^2 N_c = \text{const.}, \ N_c \to \infty$ (Nucl.Phys.B160 (1974) 57 E. Witten)

- Planar type meson current correlator dominates → saturated by `confined meson' state

- Baryon ground state

$$H = \mathcal{N} \left[ M + \int \frac{d^3x \nabla \phi^* \nabla \phi}{2M} - \frac{1}{2} g^2 \int d^3x \ d^3y \frac{\phi^*(x) \phi^*(y) \phi(x) \phi(y)}{|x - y|} - \epsilon \int d^3x \phi^* \phi(x) \right]$$

$+ O(N_c)$ interaction term in medium
Large $N_c$ QCD and quarkyonic matter

- $g_T^2 N_c = \text{const.}, \ N_c \to \infty$ \hspace{1em} \cite{McLerran2007}

  \[ \epsilon \sim \frac{k_F^2}{2M} \]

  (Nonrelativistic expansion)

  \[ N_c (\psi \bar{\psi})^2 / \Lambda_{QCD}^2 \]

  (Baryon interaction)

- Phase diagram

- Quarkyonic matter: EOS in order of $N_c$

  \[ k_F \sim \frac{1}{N_c} \Lambda_{QCD} \]

  dilute gas-quarkyonic window

  \[ P_{\text{pert.}}(\mu) \sim N_c N_f \mu^4 F_0 \left( g^2 (\mu / \Lambda_{QCD}), N_f \right) \]

  perturbative quark matter EOS

- Low energy excitation on the quark Fermi surfaces would be baryons
  \rightarrow baryon interaction will determine low-T matter properties
Large $N_C$ theory and Quarkyonic matter

- Quark Fermi sea cannot screen gluon

$$\Pi_{\mu\nu}^{ab}(Q) = g^2\delta^{ab} \int \frac{d^4K}{(2\pi)^4} \text{Tr} [\gamma_\mu S_F(K)\gamma_\nu S_F(K - Q)]$$

$$= m^2\delta^{ab} \int \frac{d\Omega}{4\pi} \left( \delta_{\mu_4\delta_{\nu_4}} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q \cdot K} \right)$$

$$m^2 = \frac{1}{3} g^2 T^2 \left( C_A + \frac{1}{2} n_f \right) + \frac{1}{2} g^2 \sum_j \frac{\mu_j^2}{\pi^2} \sim \frac{1}{N_C}$$

$g_T^2 N_C = \text{const.}$

Quarks on the Fermi surface $\rightarrow$ confined as done in vacuum confinement mechanism

- A lot of rooms for quarks in $\Lambda_{QCD}$ ball

(Figure from PRL122(2019)122701)
Broadly distributed quarks

- Quarks in $\Lambda_{QCD}$ ball can broadly distributed ($k^F_N < \Lambda_{QCD}$)

(Figure from PRL122(2019)122701)
Fast enough nucleon-like shell

- Once quark Fermi sea is formed \( (k^F_N \sim \Lambda_{QCD}) \)

\[
k^F_N = N_c k^F_q > \Lambda_{QCD}
\]

- There is no sudden jump of total baryon number density

\[
n_B^{\text{tot}} = \frac{2N_f}{3\pi^2} k^F_q
\]

- If one assumes constituent quark model \( (m_q \sim \Lambda_{QCD}) \), \( \mu_N \) grows as

\[
\mu_N = \sqrt{(N_c k^F_q)^2 + m_N^2}
\]

- Total energy density is kept

\[
\epsilon_q \sim \Lambda_{QCD} n_q
\]

\[
\sim N_c \Lambda_{QCD}^4 = M_N \Lambda_{QCD}^3 \sim \epsilon_N
\]

\[
\rightarrow \text{ pressure is enhanced (stiff EOS) } \quad p = -\epsilon + \mu n
\]

\((\epsilon, n \text{ are kept in similar order})\)

Detailed explanations are given in many literatures Nucl.Phys.A796.83, arXiv:1904.05080, etc.
Phenomenological approaches

• $N_c \to \infty$ limit contains intrinsic scale ($\Lambda_{QCD}$) and divergence ($N_c$)

• Like Hagedorn model

\[ \lim_{E \to \infty} \rho(E) dE = \frac{a}{E^{5/2}} e^{E/T_0} dE \]

\[ \lim_{T \to T_0} E \sim \alpha \frac{T_0^2}{T_0 - T} \]

Density distribution measure for high energy state

→ leads to singular entropy $S \sim |T - T_0|^{-\alpha}$

Near the critical temperature, the energy diverges

→ system prefer to make new degree of freedom

• If one tries singular $\mu$ near critical density $n_0$ and quark d.o.f.

\[ \gamma = 0.7 \]

\[ n_0 \sim 4 \rho_0 \]

\[ \mu_N = M + \kappa \frac{M}{N_c^2} \left\{ (1 - n_N^n/n_0)^{-\gamma} - 1 \right\} \]

\[ \nu_s^2 = \frac{n_N^n + n_Q^n}{\mu_N (dn_N^n/d\mu_N + dn_Q^n/d\mu_N)} \]

Sudden increment of $\mu$ with onset of quarks and recovery of pQCD limit by large number of quarks

Hard-core repulsion by “effective size”

• To be more realistic

Lattice QCD study for hadron interaction (HALQCD T. Hatsuda et al.)

• Hard core nature can be embodied by semi-classical size $v_0$

\[ V_{ex} = V \left(1 - \frac{n}{n_0}\right) \]
\[ n_0 = \frac{1}{v_0} \]
\[ n_{ex}^b = \frac{n_b}{1 - n_b/n_0} \]
\[ = \frac{2}{(2\pi)^3} \int_0^{K_b^0} d^3k \]

Hardcore effective size and excluded volume
→ reduced available space (fast nucleons)
Quarkyonic-like baryon shell structure

• Considering Pauli exclusion principle ($\mu_N = N_c \mu_q$)

\[
\tilde{z} = 4 \left( 1 - \frac{n_N^N}{n_0} \right) \int_{k_F}^{k_F + \Delta} d^3k \frac{e^{k^2}}{(2\pi)^3} \left( (N_c m_Q)^2 + k^2 \right)^{1/2} + \frac{2Nc}{\pi^2} \int_{0}^{k_F/N_c} dk k \left( \Lambda^2 + k^2 \right)^{1/2} \left( m_Q^2 + k^2 \right)^{1/2}
\]

- Quark density is determined by minimization of energy density
- IR cutoff $\Lambda$ is introduced to regularize singularity (artifact) near onset of quark

\[
\tilde{n}_Q^N = \frac{2}{3\pi^2} \left( (k_Q^2 + \Lambda^2)^{3/2} - \Lambda^3 \right)
\]

- Appearance of quarks makes the stiff increment of chemical potential

![Graphs showing n_Q/n_B for different n_0 and \Lambda values](image-url)
Quarkyonic-like baryon shell structure

- After tuning hard-core density to match energy density scale

- Hardcore repulsive interaction reproduces the EOS deduced from Quarkyonic picture

- What would be happening if we consider the EM charge and strangeness?
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Three-flavor excluded-volume model

- Effective sizes and Pauli principle

\[
\tilde{n}_{b_i} = \frac{n_{b_i}}{1 - \tilde{n}_b / n_0} = \frac{2}{(2\pi)^3} \int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}} d^3k,
\]

\[
\tilde{n}_b = n_n + n_p + (1 + \alpha)n_\Lambda,
\]

\(\alpha\) determines relative hard-core size of \(\Lambda\) hyperon

- Quark measure and energy density

\[
n_{Q_i} = \frac{1}{\frac{1}{2}} \int_0^{k_F^{Q_i}} dk \left( k^2 + \Lambda_{Q_i}^2 \right) = \frac{k_F^{Q_i}}{3 \pi^2} \left( 1 + \frac{3}{2} \left( \Lambda_{Q_i} / k_F^{Q_i} \right)^2 \right)
\]

Non-perturbative cutoff scale \(\Lambda_Q\)

\[
\epsilon_{QY} = 2 \left( 1 - \frac{\tilde{n}_b}{n_0} \right) \sum_i \int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}} \frac{d^3k}{(2\pi)^3} \left( k^2 + m_{b_i}^2 \right)^{1/2} + \frac{N_c}{\pi^2} \sum_{u,d,s} \int_{k_F^{Q_j}}^{k_F^{Q_j}} dk \left( k^2 + \Lambda_{Q_j}^2 \right) \left( k^2 + m_{Q_j}^2 \right)^{1/2} + \frac{(3\pi^2)^{\frac{3}{2}}}{4\pi^2} n_e^4,
\]

Explicit shell-like distribution in phase space and electro-magnetic charge
Electromagnetic charge and strangeness

- Baryon number conservation

\[
\begin{align*}
    n_B &= n_B + n_Q, \\
    dn_B &= dn_B + dn_Q = dn_n + dn_p + dn_{\Lambda} + dn_{\bar{u}} + dn_{\bar{d}} + dn_{\bar{s}} = 0,
\end{align*}
\]

- EM charge neutrality and \textit{weak}-equilibrium

\[
\begin{align*}
    n_e &= n_p + 2n_{\bar{u}} - n_{\bar{d}} - n_{\bar{s}}, \\
    dn_e &= dn_p + 2dn_{\bar{u}} - dn_{\bar{d}} - dn_{\bar{s}}.
\end{align*}
\]

\[
\begin{align*}
    \mu_n &= \mu_p + \mu_e, \\
    \mu_{\bar{d}} &= \mu_{\bar{u}} + 3\mu_e, \\
    \mu_n &= \mu_{\Lambda} \ (\text{when } n_{\Lambda} \neq 0, \ n_{\Lambda} = 0 \ \text{if } \mu_{\Lambda} < m_{\Lambda}), \\
    \mu_{\bar{d}} &= \mu_{\bar{s}} \ (\text{when } n_{\bar{s}} \neq 0, \ n_{\bar{s}} = 0 \ \text{if } \mu_{\bar{s}} < N_c m_s).
\end{align*}
\]

Tilde represents unit in baryon number

- Minimization of energy density

\[
\begin{align*}
    d\varepsilon &= \mu_n dn_n + \mu_p dn_p + \mu_{\Lambda} dn_{\Lambda} + \mu_{\bar{u}} dn_{\bar{u}} + \mu_{\bar{d}} dn_{\bar{d}} + \mu_{\bar{s}} dn_{\bar{s}} + \mu_e dn_e \\
    &= \mu_n (dn_n + dn_p) + \mu_{\Lambda} dn_{\Lambda} + (\mu_{\bar{d}} - \mu_e) (dn_{\bar{d}} + dn_{\bar{s})} + (\mu_{\bar{s}} - \mu_e) dn_{\bar{s}} = 0,
\end{align*}
\]

if \( n_{\Lambda} = 0, \ n_s = 0, \ \mu_n = N_c \mu_d - \mu_e = \mu_p + \mu_e, \)
if \( n_{\Lambda} \neq 0, \ n_s \neq 0, \ \mu_n = N_c \mu_d - \mu_e = \mu_{\Lambda} = \mu_p + \mu_e = N_c \mu_s - \mu_e, \)
if \( n_{\Lambda} = 0, \ n_s \neq 0, \ \mu_n = N_c \mu_d - \mu_e = \mu_{\Lambda} = \mu_p + \mu_e, \)
if \( n_{\Lambda} \neq 0, \ n_s = 0, \ \mu_n = N_c \mu_d - \mu_e = \mu_p + \mu_e = N_c \mu_s - \mu_e. \)

\[
\mu_N = N_c \mu_q
\]

Three-flavor extension of dynamical equilibrium constraints
Effectively three-flavor matter

- **Baryon chemical potential** (with $\omega_{n,p} = 1$, $\Omega = 1 + \alpha$)

\[
\mu_{bi} = \frac{\partial \bar{\varepsilon}_{QY_i}}{\partial n_{bi}} = \left( \frac{n_0 - (\bar{n}_b - \omega_i n_{bi})}{n_0 - \bar{n}_b} \right) \left( [k_F + \Delta]_{bi}^2 + m_{bi}^2 \right)^{\frac{1}{2}} + \frac{\omega_i}{n_0} \left\{ \sum_{j \neq i} n_{bj}^{eq} \left( [k_F + \Delta]_{bj}^2 + m_{bj}^2 \right)^{\frac{1}{2}} - \sum_k \frac{1}{n_0} \int_{k_F}^{[k_F + \Delta]_{bk}} dk k^2 \left( k^2 + m_{bk}^2 \right)^{\frac{1}{2}} \right\}
\]

- $\omega_i$ dependent term can have non-zero contribution even if $n_i = 0$
  → reduced available space enhances the chemical potential $\mu_{bi}$

- If $\omega_i \simeq \omega_{n,p}$ and $m_i > m_{n,p}$
  → *the heavier state (such as $\Delta$ isobar) is hard to appear even at high density regime*

- **Quark chemical potential**

\[
\mu_{Qi} = \frac{\partial \bar{\varepsilon}_{QY_i}}{\partial n_{Qi}} = \left(1 - \frac{\bar{n}_b}{n_0}\right) \sum_k \frac{\partial k_F^{bk}}{\partial k_{Q_i}^2} \left( [k_F + \Delta]_{bk}^2 + m_{bk}^2 \right)^{\frac{1}{2}} - \left( k_{Q_i}^2 + m_{Q_i}^2 \right)^{\frac{1}{2}} + N_c \left( (k_{Q_i}^2)^2 + m_{Q_i}^2 \right)^{\frac{1}{2}}
\]

- quark Fermi sea supports the shell-like distribution of baryons
  → $\mu_{Qi}$ get enhanced by if $Q_i$ support a huge amount of baryon shell distribution
Structure of the shell-like distribution

- Strongly correlated assumption
  - If $d$ quarks are saturated first, and $u$ quarks follow
  - Confined $u$ quark momenta should be bounded around the confined $d$ quark momenta
  - All confined quarks have momentum in the similar order $k_{Q_i} \approx k_B/3$
    \[ k_{conf.}^u = k_F^d + r_{qq}^s w_s (k_F^d - k_F^u) \]
    \[ w_s(x) = 1 - \exp\left(-\frac{|x|^2}{\delta^2}\right) \]
  - Lower boundary of the shell-like distribution
    \[ k_F^n = \Theta(k_F^d - k_F^n) \left(3k_F^d + r_{qq}^s w_s (k_F^d - k_F^n)\right) \]
    \[ + \Theta(k_F^u - k_F^n) \left(3k_F^u + 2r_{qq}^s w_s (k_F^u - k_F^n)\right), \]
    \[ k_F^p = \Theta(k_F^d - k_F^p) \left(3k_F^d + 2r_{qq}^s w_s (k_F^d - k_F^p)\right) \]
    \[ + \Theta(k_F^u - k_F^p) \left(3k_F^u + r_{qq}^s w_s (k_F^u - k_F^p)\right), \]
    \[ k_F^A = \Theta(k_F^d - k_F^A) \left(3k_F^d + r_{qq}^s w_s (k_F^d - k_F^A) + r_{qs}^s w_s (k_F^d - k_F^A)\right) \]
    \[ + \Theta(k_F^u - k_F^A) \left(3k_F^u + r_{qs}^s w_s (k_F^u - k_F^A) + r_{qs}^s w_s (k_F^u - k_F^A)\right), \]

Structure of the shell-like distribution

- Weakly correlated assumption
  
- If $d$ quarks are saturated first, and $u$ quarks follow
  
- Confined $u$ quark momenta can be distributed away from the confined $d$ quark momenta
  
- $u$ quarks can have momentum $k_u \leq k_b/3$
  
- Error function can be assumed for the weight
    
    \[ k_{\text{conf.}}^u = k_F^d + r_{qq}^u w_w (k_F^d - k_F^u) \]
    
    \[ w_w(x) = \text{erf}(-|x|/\delta) \]
  
- Lower boundary of the shell-like distribution
  
  \[
  \begin{align*}
  k_F^n &= \Theta(k_F^d - k_F^u) \left( 3k_F^d + r_{qq}^u w_w (k_F^d - k_F^u) \right) \\
  &\quad + \Theta(k_F^d - k_F^u) \left( 3k_F^d + 2r_{qq}^u w_w (k_F^d - k_F^u) \right), \\
  k_F^p &= \Theta(k_F^d - k_F^u) \left( 3k_F^d + 2r_{qq}^u w_w (k_F^d - k_F^u) \right) \\
  &\quad + \Theta(k_F^d - k_F^u) \left( 3k_F^d + r_{qq}^u w_w (k_F^d - k_F^u) \right), \\
  k_F^\wedge &= \Theta(k_F^d - k_F^u) \left( 3k_F^d + r_{qq}^u w_w (k_F^d - k_F^u) + r_{qs}^u w_s (k_F^d - k_F^u) \right) \\
  &\quad + \Theta(k_F^d - k_F^u) \left( 3k_F^d + r_{qs}^u w_s (k_F^d - k_F^u) + r_{qs}^u w_s (k_F^d - k_F^u) \right),
  \end{align*}
  \]
Density profile of quasi-particles

- Strongly correlated assumption with $\alpha > 0$ and $\alpha < 0$

- Larger repulsive core of $\Lambda$ ($\alpha > 0$) leads to $n_A = 0$

- If there are baryon shells supported by the quark Fermi sea, the corresponding quark Fermi sea becomes flavor symmetric
  → asymmetric configuration causes huge enhancement of $\mu_{b_i}$
  → dynamically unfavored
Density profile of quasi-particles

- Weakly correlated assumption with $\alpha > 0$ and $\alpha < 0$

- Larger repulsive core of $\Lambda$ ($\alpha > 0$) leads to $n_\Lambda = 0$

- Even if there are baryon shells supported by the quark Fermi sea, the asymmetric configuration of the quark Fermi sea is appearing
  → asymmetric configuration does not cause that huge enhancement of $\mu_{b_i}$
  → dynamically allowed
Hard-soft evolution of EOS

Strongly correlated

Weakly correlated

\( p_{\text{B}} \) [MeV/fm\(^3\)]

\( \varepsilon_{\text{eq}} \) [MeV/fm\(^3\)]

\( c_s^2 \)

\( n_B/\rho_0 \)
Mass-radius relation of stable neutron star

- With aid of Urbana IX forces \( (n_0 = 5\rho_0) \)

- Urbana IX model (for neutron rich matter)

\[
E/A = \left(p_F^2 + m_n^2\right)^{\frac{1}{2}} - m_n + \tilde{a} \left(\frac{n_n}{\rho_0}\right) + \tilde{b} \left(\frac{n_n}{\rho_0}\right)^2, \quad \tilde{a} = -28.3 \text{ MeV}, \quad \tilde{b} = 10.7 \text{ MeV}
\]

(chosen by requiring minimal Maxwell construction interval)

- \( M_{\text{max}} = 2.03M_\odot, \; R_{1.4} = 12.5 \text{ km}, \; R_{2.03} = 11.4 \text{ km} \)

- \( n_{\bar{Q}} = 0.26 \; n_B, \; \varepsilon_{\bar{Q}} = 0.27 \; \varepsilon_{\text{tot}} \) at the core of maximum mass state
Some similar studies

- Chiral effective theory and pQCD limits


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Summary

- Neutron star puzzle may be solved via quarkyonic matter concept

- Repulsive hard-core and excluded-volume model contains dynamically saturated quark Fermi sea and shell-like distributions

- Strangeness does not cause any problem!
  (actually, it is needed to reach the massive state)

- **What is the true nature of the non-perturbative state on the quark Fermi surface?**