Effective range corrections from effective field theory with di-baryon fields and perturbative pions

Shung-Ichi Ando

Daegu University

in collaboration with C. H. Hyun (Daegu U.),

Outline

1. Introduction:
   Why di-baryon fields ?
   Why (and how) perturbative pions ?
2. S-wave NN scattering at low energies in EFT with di-baryon fields and perturbative pions and effective ranges corrections, v_2, v_3, v_4, in higher order
3. Results and discussion

1. Introduction: Why di-baryon fields ?

- Very small scales, $a_s = -23.7$ fm and deuteron binding energy $B = 2.22$ MeV, compared to pion mass $m_\pi = 140$ MeV appear in two nucleon sector. "*Fine turning*"
- In RG analysis of NN scattering (Birse, MacGovern, Richardson): Trivial fixed point: no interaction limit

$$V = C_0 + C_2(p'^2 + p^2) + \cdots \ .$$

Non-trivial fixed point: $a \to \infty$ or $B \to 0$ limit

$$\frac{1}{V} = C_0' + C_2'(p'^2 + p^2) + \cdots \ .$$

Di-baryon formalism

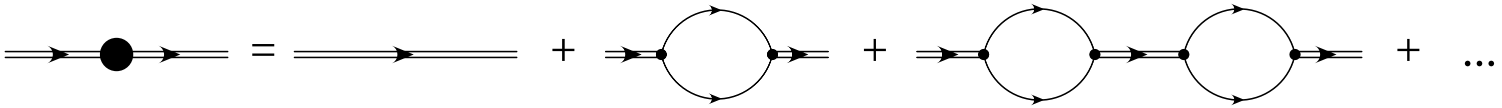
Good for expansion around the non-trivial fixed point.

- Lagrangian

$$\mathcal{L}_s = \sigma_s s_a^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2-\partial^2}{4m_N} + \Delta_s \right] s_a - y_s \left[s_a^\dagger (N^T P_a^{(^1S_0)} N) + \text{h.c.} \right],$$

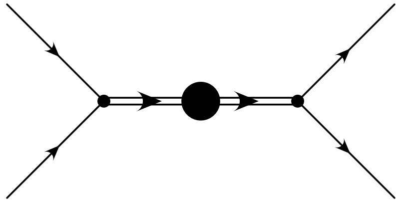
$$\mathcal{L}_t = \sigma_t t_i^\dagger \left[iv\cdot\partial + \frac{(v\cdot\partial)^2-\partial^2}{4m_N} + \Delta_t \right] t_i - y_t \left[t_i^\dagger (N^T P_i^{(^3S_1)} N) + \text{h.c.} \right],$$

- Dressed di-baryon propagator



$$iD_{s,t}^{-1}(p) = iy_{s,t}^2 \frac{m_N}{4\pi} \left[\frac{4\pi\sigma_{s,t}\Delta_{s,t}}{m_N y_{s,t}^2} + \frac{4\pi\sigma_{s,t}}{m_N^2 y_{s,t}^2} p^2 + \mu + ip \right],$$

- Scattering amplitude



$$iA_{s,t} = \frac{4\pi}{m_N}\frac{i}{-\mu - \frac{4\pi\sigma_{s,t}\Delta_{s,t}}{m_N y^2} - \frac{4\pi\sigma_{s,t}}{m_N^2 y_{s,t}^2}p^2 - ip}.$$

- Renormalization with effective range expansion:

$$iA = \frac{m_N}{4\pi}\frac{1}{p\cot\delta_0 - ip}, \quad p\cot\delta_0 = -\frac{1}{a} + \frac{1}{2}rp^2 + \cdots,$$

$$-\frac{1}{a} = -\mu - \frac{4\pi\sigma_{s,t}\Delta_{s,t}}{m_N y_{s,t}^2}, \quad r = -\frac{8\pi\sigma_{s,t}}{m_N^2 y_{s,t}^2},$$

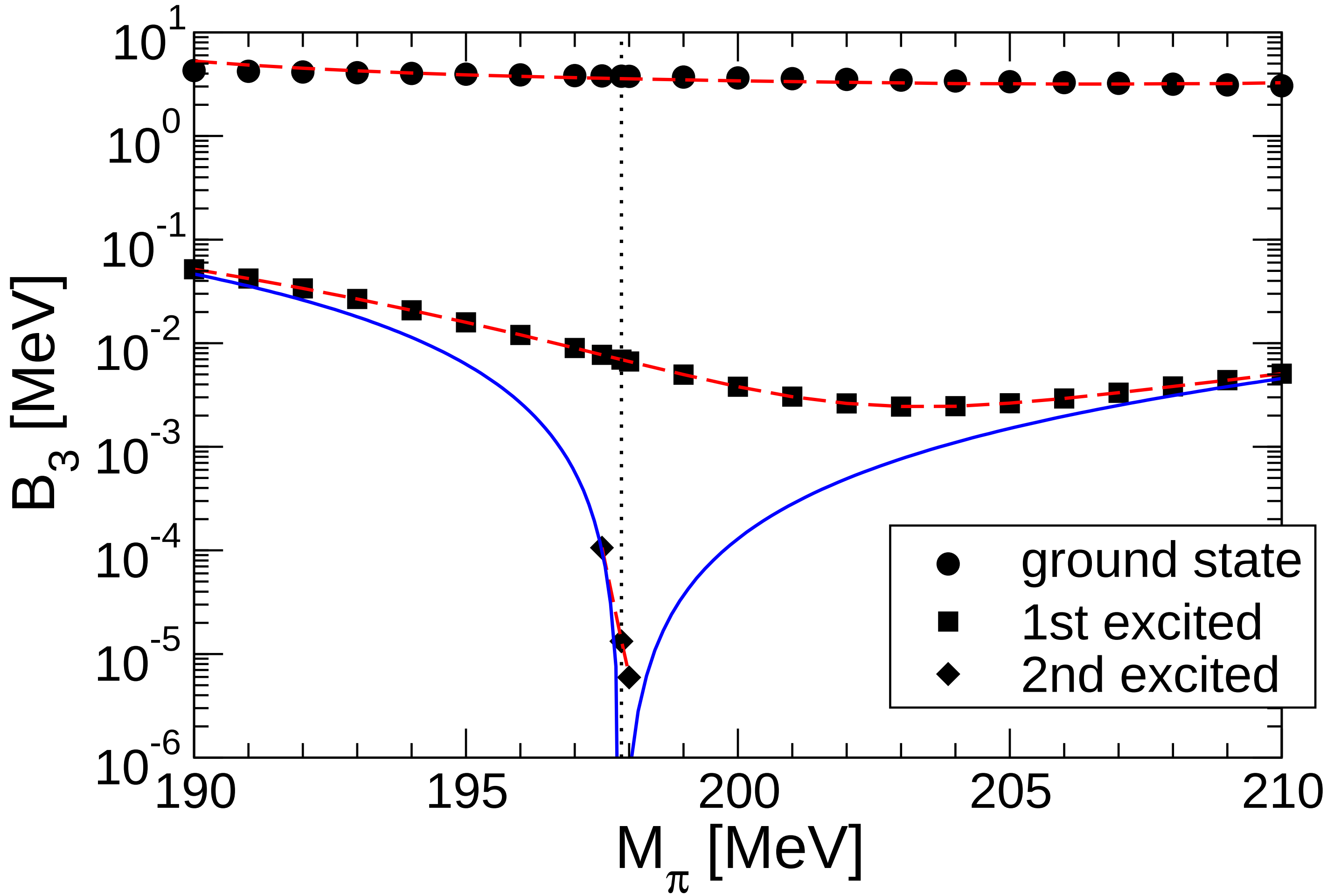
Why (and how) perturbative pions ?

- Whether the pions are perturbative or not depends on the scale.
- Some values of the pion mass may be worth thinking again:

$$m_\pi = 0\,, \quad 140\,, \quad 198\,, \quad 354\,\mathrm{MeV}\,.$$

- The critical mass in the unitary limit, $m_\pi^{crit} = 198$ MeV where $a = \infty$ and $B_2 = 0$ MeV. The three nucleon system becomes universal and has "Efimov states",

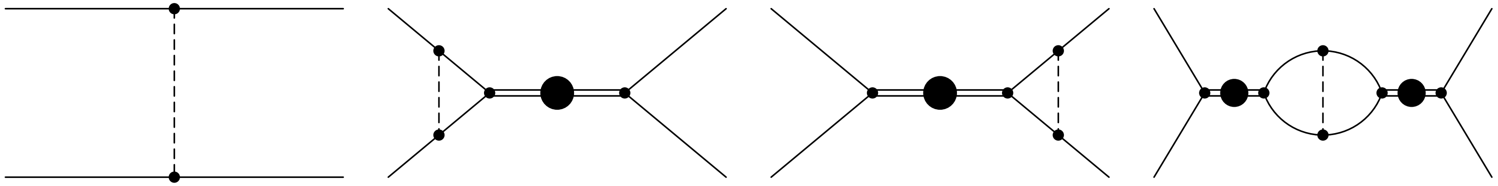
$$B_3^{(n)} = \left(e^{2\pi/s_0}\right)^{1-n} B_3^{(1)} \simeq (515)^{1-n} B_3^{(1)}\,.$$



E. Epelbaum et al. Eur. Phys. J. C 48, 169-178 (2006).

2. S-wave NN scattering in the EFT

- Pions are included perturbatively around the non-trivial fixed point by utilizing the di-baryon fields.
- Diagrams



(I) (II-a) (II-b) (III)

$$
\begin{aligned}
A &= A_d + A_\pi\,, \\
A_\pi &= A_{(I)} + A_{(II)} + A_{(III)}\,, \\
p\cot\delta_0 &= ip + \frac{4\pi}{m_N}\frac{1}{A} = ip + \frac{4\pi}{m_N}\frac{1}{A_d + A_\pi} \simeq ip + \frac{4\pi}{m_N}\frac{1}{A_d} - \frac{4\pi}{m_N}\frac{A_\pi}{A_d^2}
\end{aligned}
$$

The OPE contributions have been obtained by Kaplan, Savage, Wise in DR, PDS scheme.

• Results

$$p\cot\delta_0 = -\frac{1}{a} + \frac{1}{2} r p^2 + F(p)\,,$$

where

$$-\frac{1}{a_d(\mu)} \equiv -\mu - \frac{4\pi\sigma\Delta}{m_N y^2} = \left\{ -\frac{1}{a} + \frac{g_A^2 m_N}{16\pi f_\pi^2}\left[-\mu^2 + m_\pi^2 + m_\pi^2 \ln\left(\frac{\mu}{m_\pi}\right)\right]\right\} \times \left[1 - \frac{g_A^2 m_N}{8\pi f_\pi^2}(-\mu + m_\pi)\right]^{-1},$$

$$\frac{1}{2} r_d(\mu) \equiv -\frac{4\pi\sigma}{m_N^2 y^2} = \left\{ \frac{1}{2} r + \frac{g_A^2 m_N}{16\pi f_\pi^2}\left[-1 + \frac{8}{3}\frac{1}{m_\pi a_d(\mu)} - \frac{2}{m_\pi^2 a_d^2(\mu)}\right]\right\} \times \left[1 - \frac{g_A^2 m_N}{8\pi f_\pi^2}(-\mu + m_\pi)\right]^{-1},$$

$$F(p) = -\frac{g_A^2 m_N}{16\pi f_\pi^2}\left\{ \left[-\frac{8}{3}\frac{1}{m_\pi a_d(\mu)} + \frac{2}{m_\pi^2 a_d^2(\mu)}\right] p^2 + 2\left(-\frac{1}{a_d(\mu)} + \frac{1}{2} r_d(\mu) p^2\right)\left[-m_\pi + \frac{m_\pi^2}{2p}\arctan\left(\frac{2p}{m_\pi}\right)\right] + \left[\left(-\frac{1}{a_d(\mu)} + \frac{1}{2} r_d(\mu) p^2\right)^2 - p^2\right]\left[-1 + \frac{m_\pi^2}{4p^2}\ln\left(1 + \frac{4p^2}{m_\pi^2}\right)\right]\right\}.$$

- Renormalization

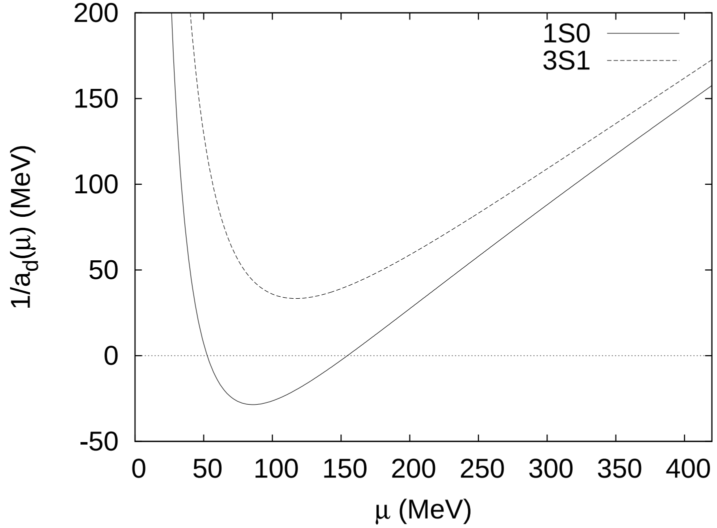
$$\frac{1}{a_0} = -8.30 \text{ (MeV)}\,, \quad r_0 = 2.73 \text{ (fm)}\,,$$

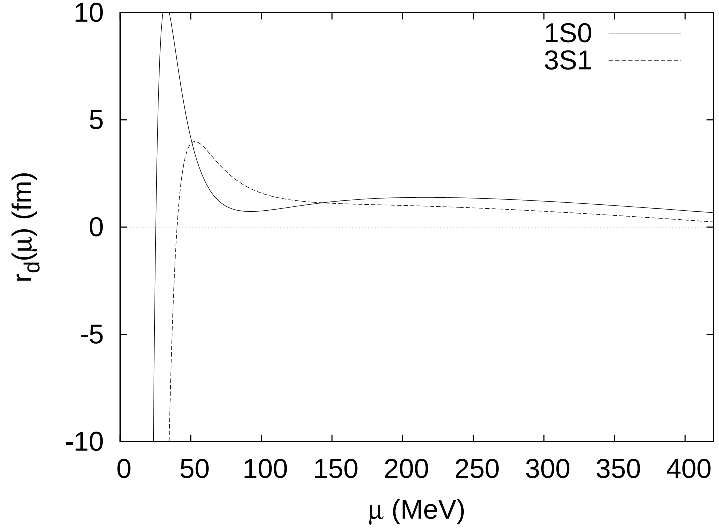
$$\frac{1}{a_1} = 36.4 \text{ (MeV)}\,, \quad r_1 = 1.76 \text{ (fm)}\,,$$

Conventionally, the scale parameter μ has been chosen as $\mu = m_\pi$. With this particular choice of the scale parameter μ, we have

$$\frac{1}{a_d(\mu)} = \frac{1}{a_0}\,, \frac{1}{a_1}\,,$$

- μ-dependence of $1/a_d(\mu)$ and $r_d(\mu)$





- Effective range parameters, v_2, v_3, v_4 in higher order

$$p\ \cot\delta = -\frac{1}{a} + \frac{1}{2}r_0p^2 + v_2p^4 + v_3p^6 + v_4p^8 + \cdots\ ,$$

where

$$v_2 = \frac{g_A^2 m_N}{16\pi f_\pi^2}\left\{-\frac{16}{3}\frac{1}{a_d^2(\mu)m_\pi^4} + \frac{32}{5}\frac{1}{a_d(\mu)m_\pi^3} - \frac{2}{m_\pi^2}\left[1+\frac{r_d(\mu)}{a_d(\mu)}\right] + \frac{4}{3}\frac{r_d(\mu)}{m_\pi}\right\},$$

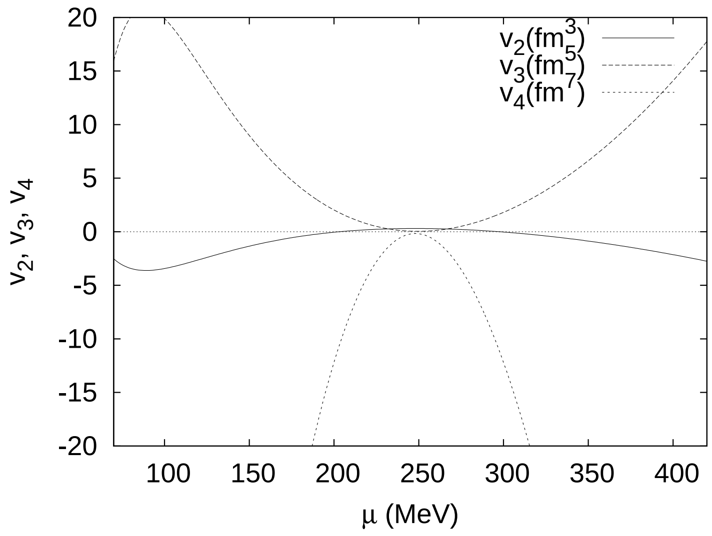
$$v_3 = \frac{g_A^2 m_N}{16\pi f_\pi^2}\left\{16\frac{1}{a_d^2(\mu)m_\pi^6} - \frac{128}{7}\frac{1}{a_d(\mu)m_\pi^5} + \frac{16}{3}\frac{1}{m_\pi^4}\left[1+\frac{r_d(\mu)}{a_d(\mu)}\right]\right.$$
$$\left. -\frac{16}{5}\frac{r_d(\mu)}{m_\pi^3} + \frac{1}{2}\frac{r_d^2(\mu)}{m_\pi^2}\right\},$$

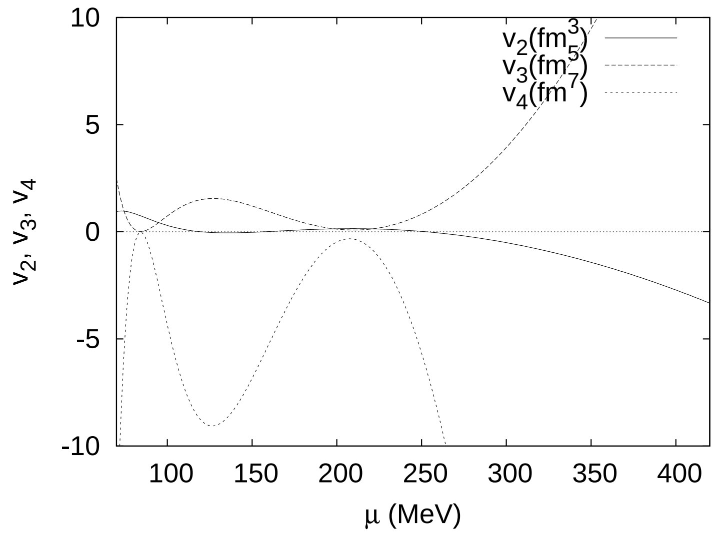
$$v_4 = \frac{g_A^2 m_N}{16\pi f_\pi^2}\left\{-\frac{256}{5}\frac{1}{a_d^2(\mu)m_\pi^8} + \frac{512}{9}\frac{1}{a_d(\mu)m_\pi^7} - 16\frac{1}{m_\pi^6}\left[1+\frac{r_d(\mu)}{a_d(\mu)}\right]\right.$$
$$\left. +\frac{64}{7}\frac{r_d(\mu)}{m_\pi^5} - \frac{4}{3}\frac{r_d^2(\mu)}{m_\pi^4}\right\}.$$

Expressions of v_2, v_3, v_4 without $r_d(\mu)$ have been obtained by Cohen and Hansen.

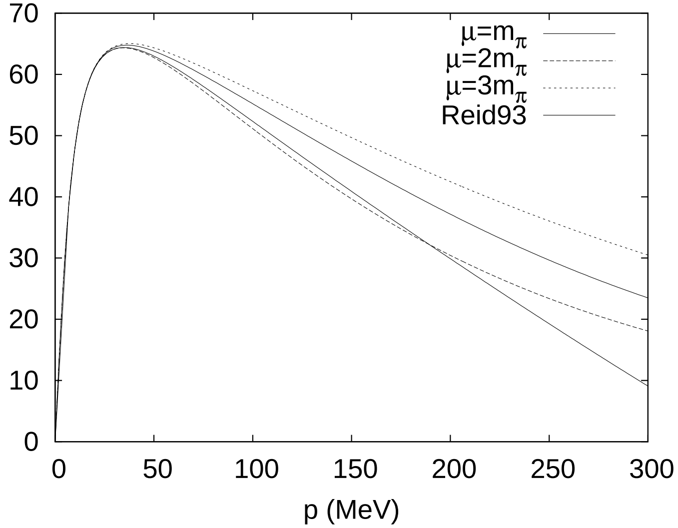
		v_2(fm^3)	v_3(fm^5)	v_4(fm^7)
$^1S_0$	PWA	-0.48	3.8	-17.0
	$a^{phy.}$, $r=0$	-3.32	17.8	-106.8
	$a^{phy.}$, $r^{phy.}$	0.69	3.99	-25.9
	$\mu = m_\pi$	-1.74	11.1	-68.0
	$\mu = 2m_\pi$	0.17	0.71	-4.85
	$\mu = 3m_\pi$	-2.75	17.8	-117
	$\mu = 178$ MeV	-0.48^*	4.40	-26.8
	$\mu = 330$ MeV	-0.48^*	4.37	-29.0
$^3S_1$	PWA	0.04	0.67	-4.0
	$a^{phy.}$, $r=0$	-0.96	4.57	-25.5
	$a^{phy.}$, $r^{phy.}$	0.44	0.48	-2.88
	$\mu = m_\pi$	-0.05	1.42	-8.22
	$\mu = 2m_\pi$	-0.25	2.39	-16.2
	$\mu = 3m_\pi$	-3.23	21.2	-139
	$\mu = 167$ MeV	0.04^*	0.75	-4.1
	$\mu = 246$ MeV	0.04^*	0.68	-4.7

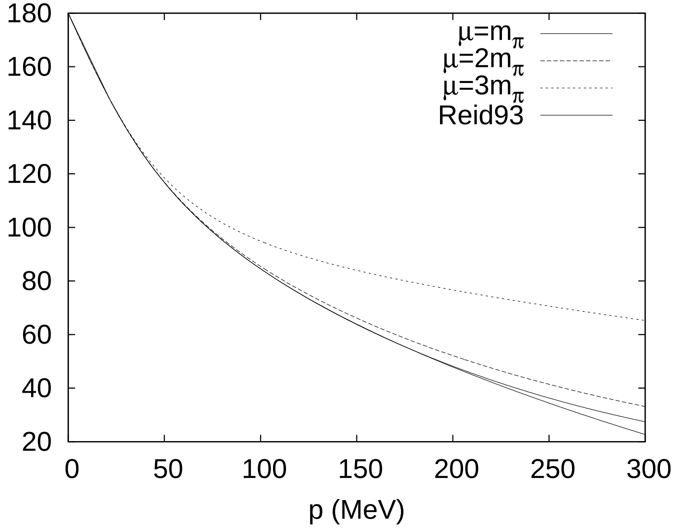
- μ-dependence of v_2, v_3, v_4

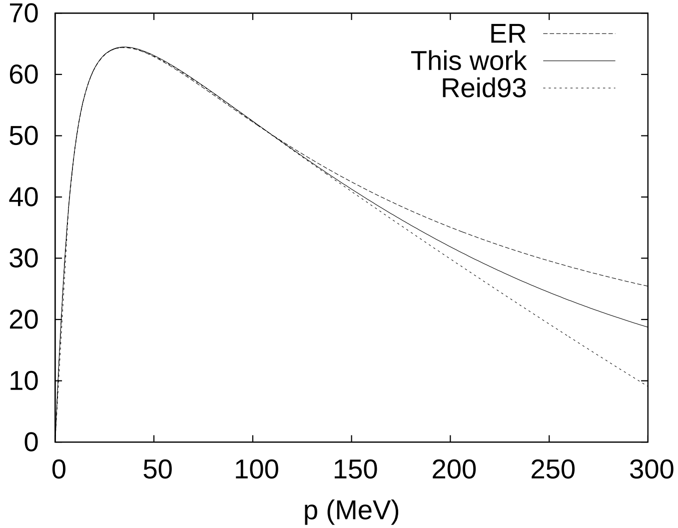


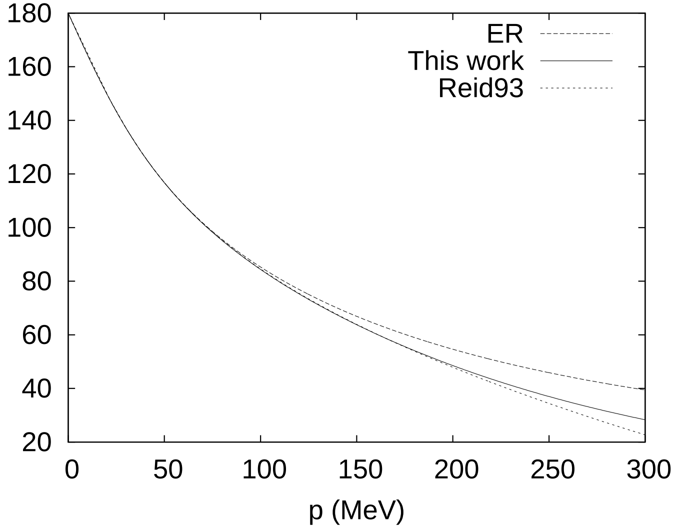


- Results: phase shifts









where $\mu = 178$ MeV ($^1S_0$) and 167 MeV ($^3S_1$).

3. Results and discussion

- EFT with di-baryon fields and perturbative pions.
- The r correction and μ dependence in v_2, v_3, v_4.
- Significant μ dependences after renormalization of $1/a$ and r.
- If μ independence is required, the theory (the phase shift) is valid up to $p \sim 50$ MeV.
- If μ is fine turned, the theory may be OK up to $p \sim m_\pi$.