Holographic equations of state and astrophysical compact objects

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YK, Chang-Hwan Lee, Ik Jae Shin, Mew-Bing Wan, arXiv:1108.6139 [hep-ph],
to appear in JHEP 2011

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Motivation

- There are many dense hQCD models, both in bottom-up and top-down
- What do EoSs from some top-down models say about our nature?
- Mass-radius relation of a neutron star, which is rather insensitive to its crust structure, might be a good litmus paper.

A slide for hQCD

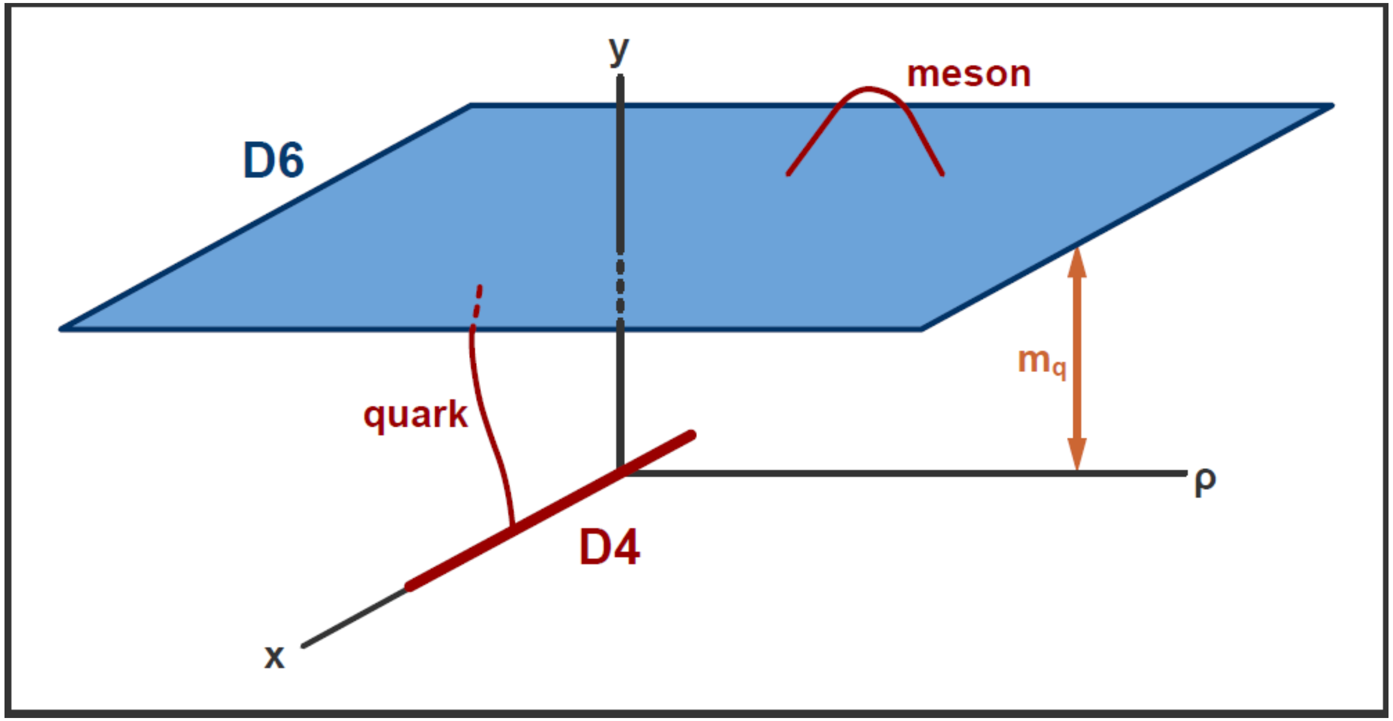
- Gluon dynamics
- (de)confinement
- Flavor (meson) dynamics
- Baryon

- Warped geometry
- Again by geometry
- Classical fields in warped geometry
- Baryon vertex (compact D-brane with N_c fundamental string attatched)

Ex: D4/D6/D6 + compact D4 model

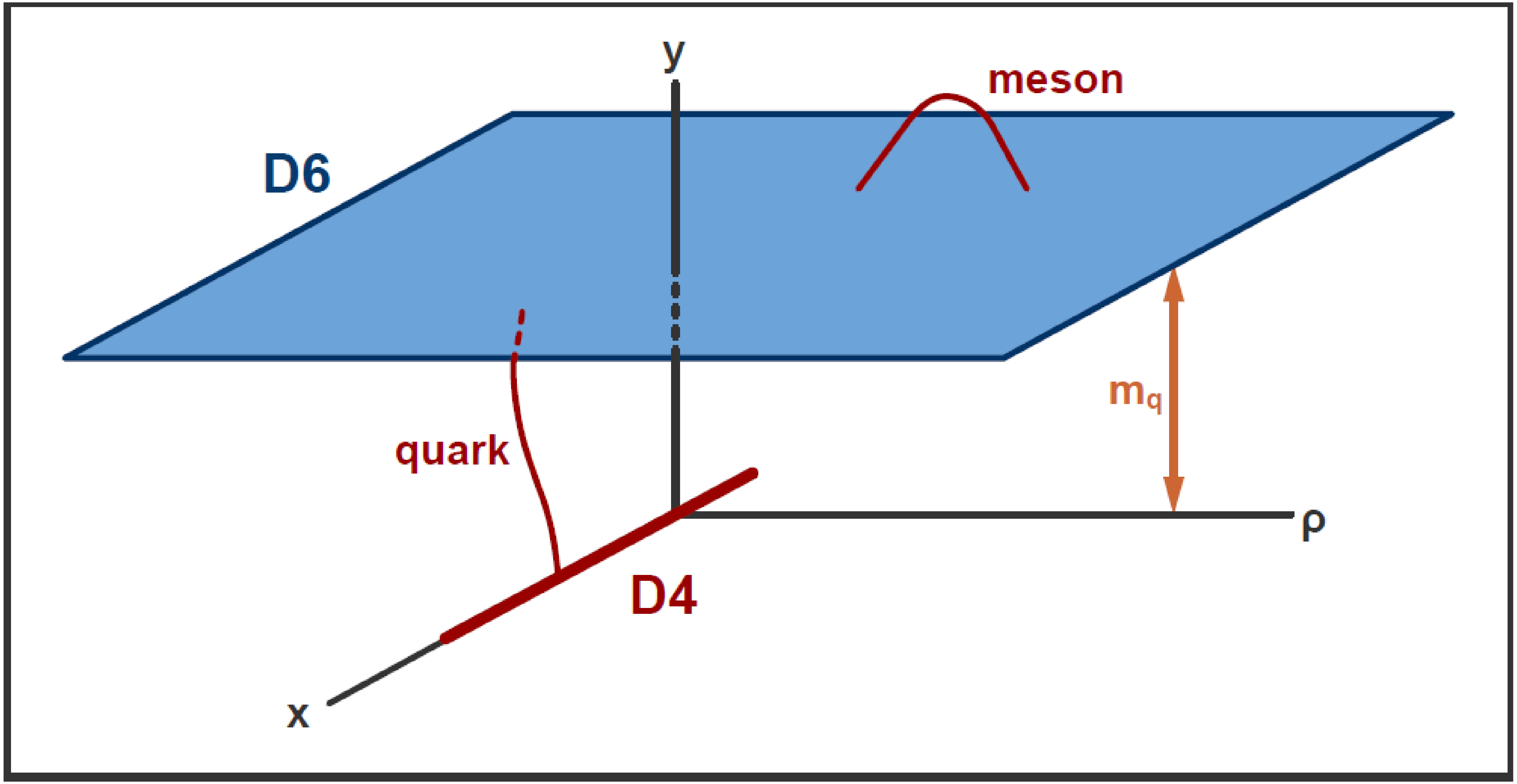
Dense D4/D6 models

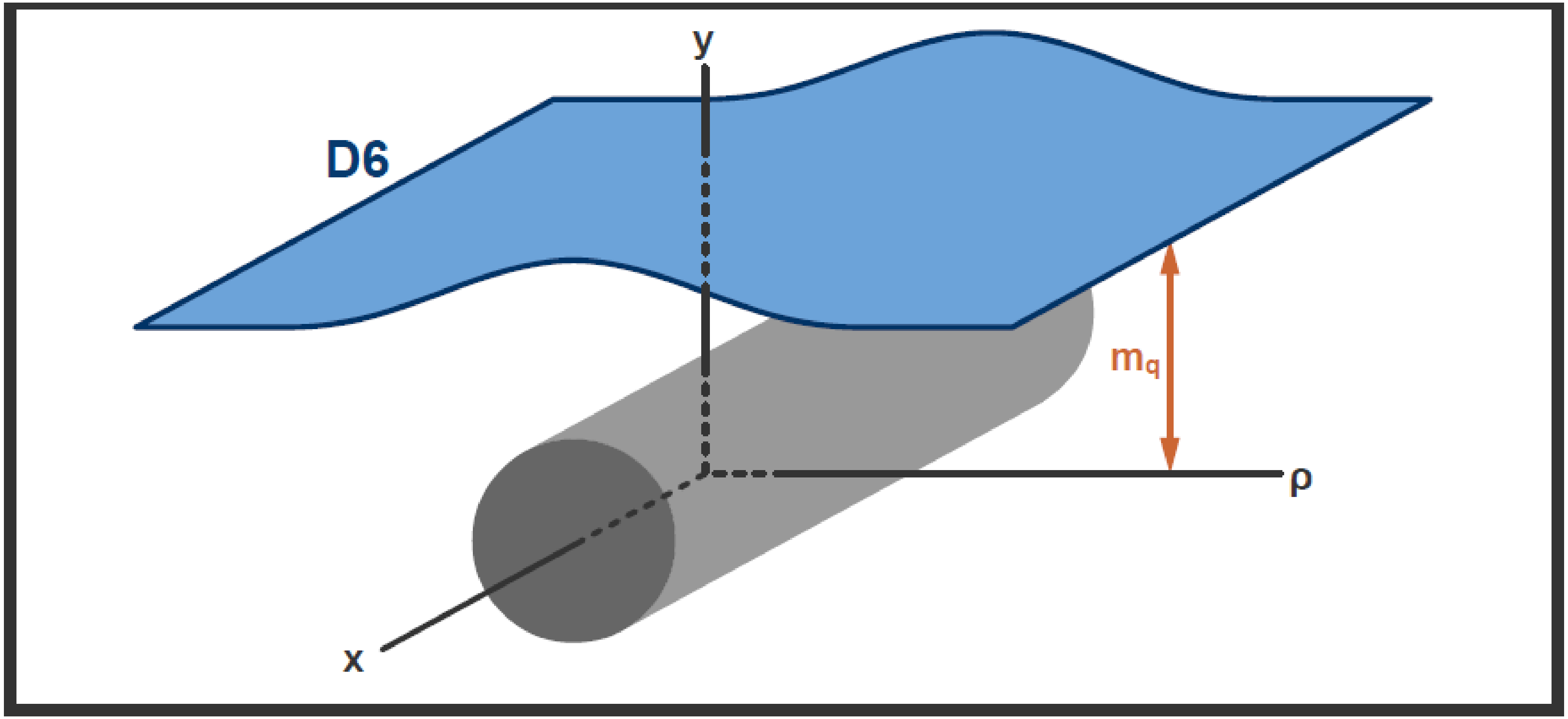
- Dense D4/D6 (Nf=1)



S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers and R. M. Thomson, JHEP **0702**, 016 (2007) [arXiv:hep-th/0611099].

Figures from Deokhyun Yi


y
meson
D6
quark
m_q
ρ
D4
x



$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(dt^2 + d\vec{x}^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{U}{\xi}\right)^2 \left(d\rho^2 + \rho^2 d\Omega_2^2 + dy^2 + y^2 d\phi^2\right),$$

where D6 brane world volume coordinates are $(t, \vec{x}, \rho, \theta_\alpha)$. The embedding ansatz is that only y depends on ρ and $\phi = 0$.

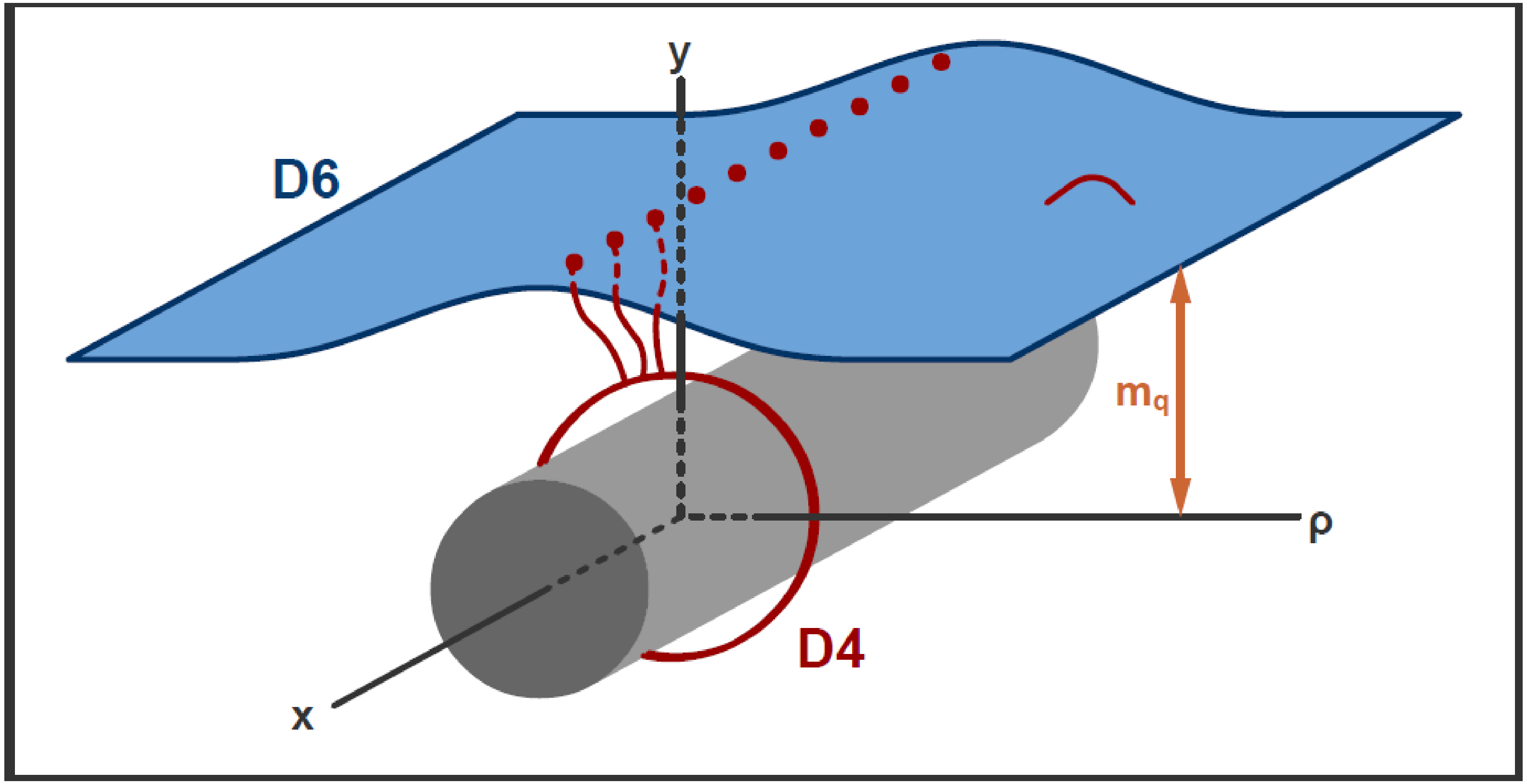
Dense hQCD

$$\text{QCD}: \ \mu_q \psi^\dagger \psi \ (= \mu_q \bar{\psi}\gamma_0 \psi) \leftrightarrow \text{Gravity}: \ V_0(x,z) = \mu_q + \cdots, \ z \to 0\,.$$

$$\text{4D generating functional}: Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}\,,$$

$$\text{5D (classical) effective action}: \Gamma_5[\phi(x,z) = \phi_0(x)]\,; \phi_0(x) = \phi(x, z=0).$$

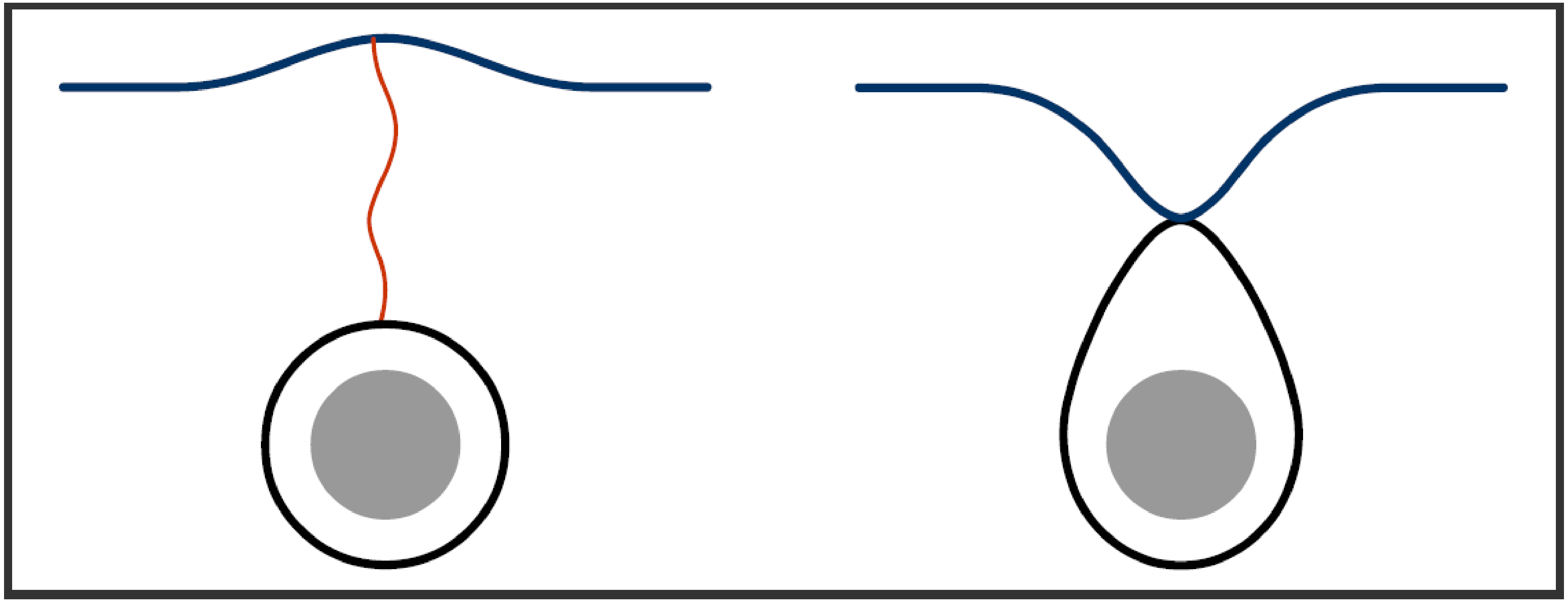
$$\text{AdS/CFT correspondence}: Z_4 = \Gamma_5.$$



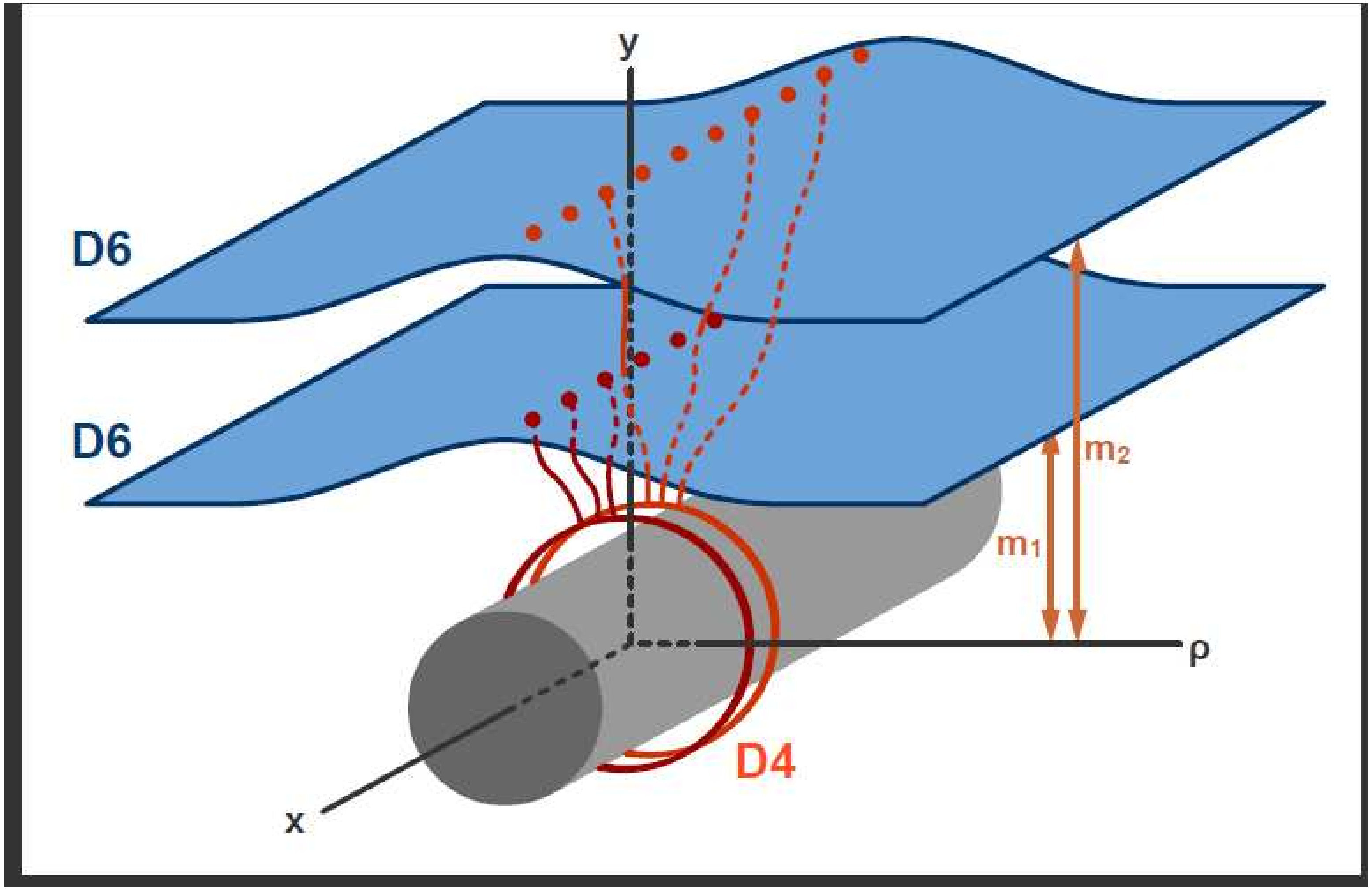
	t	1	2	3	(τ)	ξ	θ	φ_1	φ_2	φ_3
D4	•						•	•	•	•

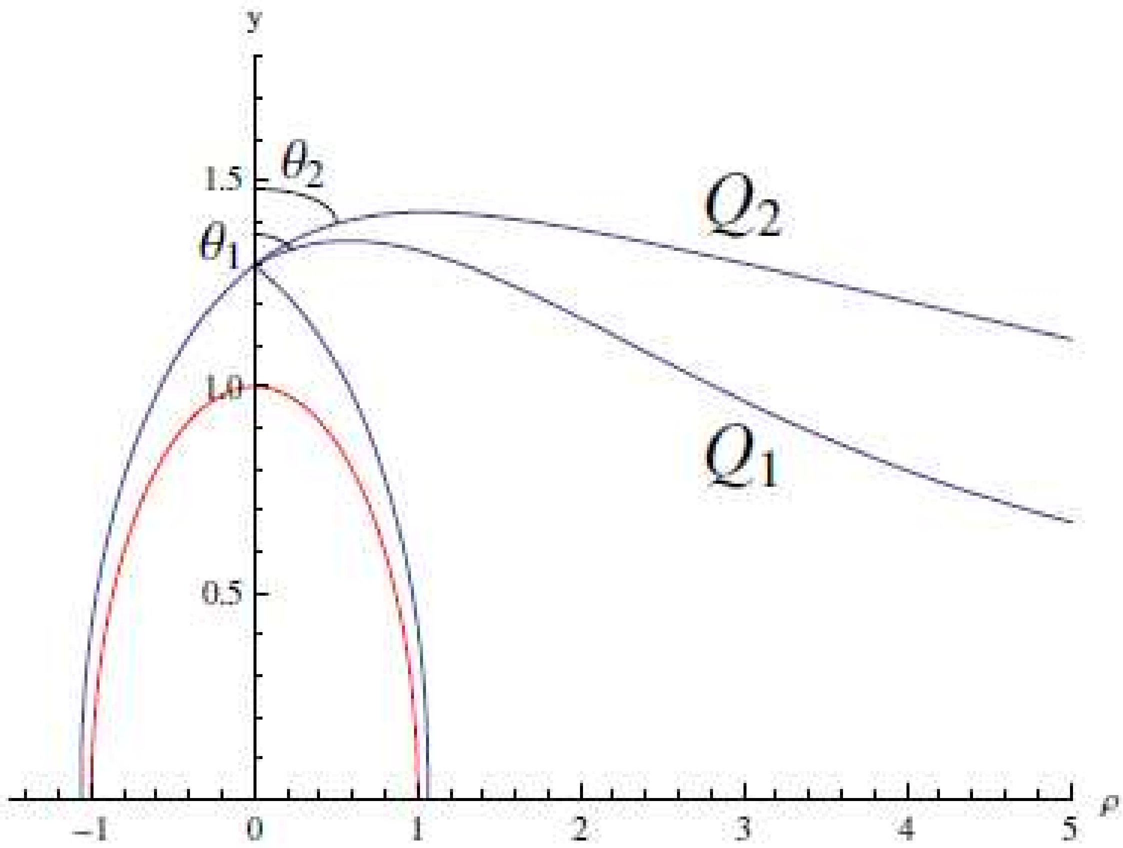
Table 2: The brane configuration : the compact D4

In holographic QCD, a compact D4 brane wrapping on the 4-sphere S^4 transverse to $\mathbb{R}^{1,3}$ is introduced as a baryon



- Dense D4/D6/D6 (Nf=2)

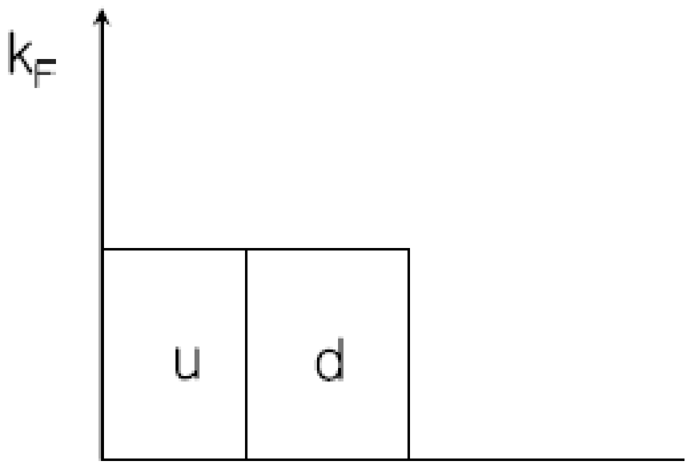


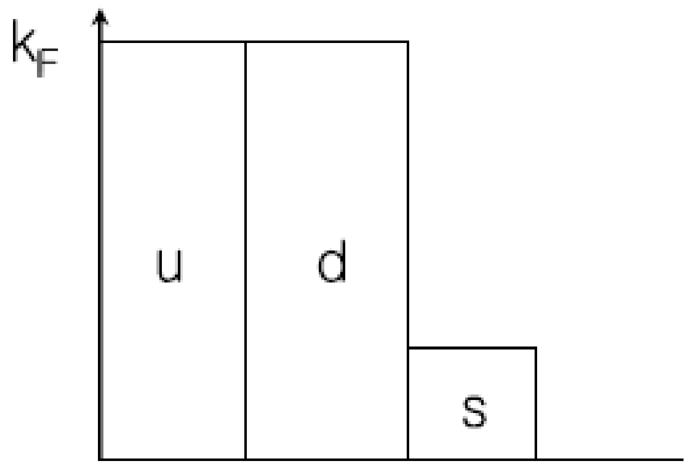


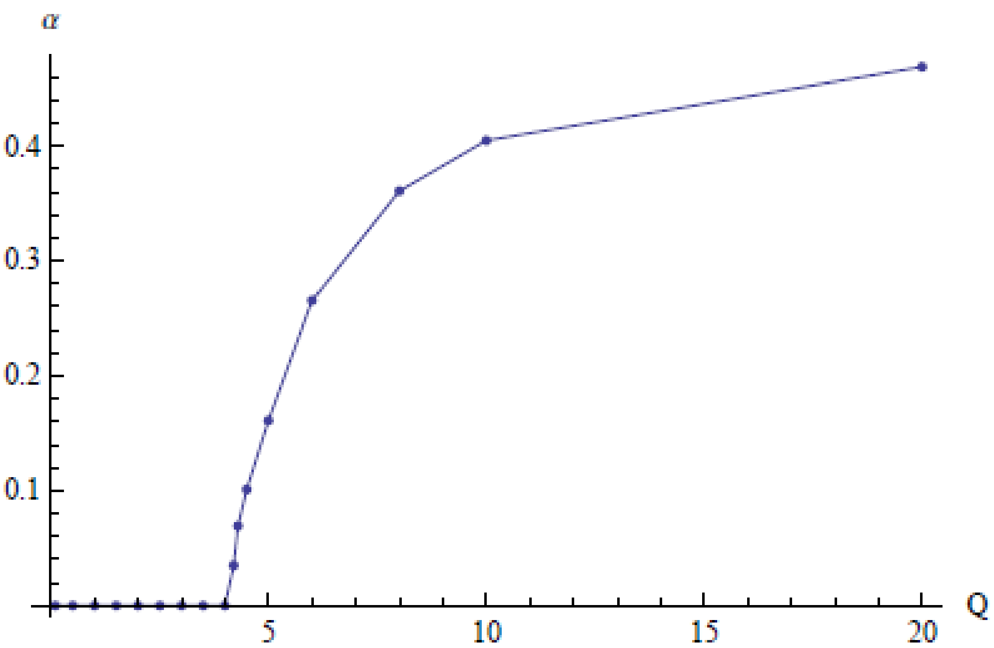
To make the system stable, following force balancing condition should be satisfied;

$$\frac{Q}{N_c} F_{D4} = F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2),$$

- Example : nuclear matter to strange matter transition







Y. Kim, Y. Seo, and S.-J. Sin, *Nuclear matter to strange matter transition in holographic QCD*, JHEP 1003 (2010) 074.

Compact stars from D4/D8 and D4/D6 models

- Tolman–Oppenheimer–Volkoff equation

$$\frac{dp}{dr} = -\frac{1}{2}(\epsilon + p)\frac{2m + 8\pi r^3 p}{r(r-2m)},$$

$$m(r) \equiv 4\pi \int_0^r \epsilon(r') r'^2 dr'$$

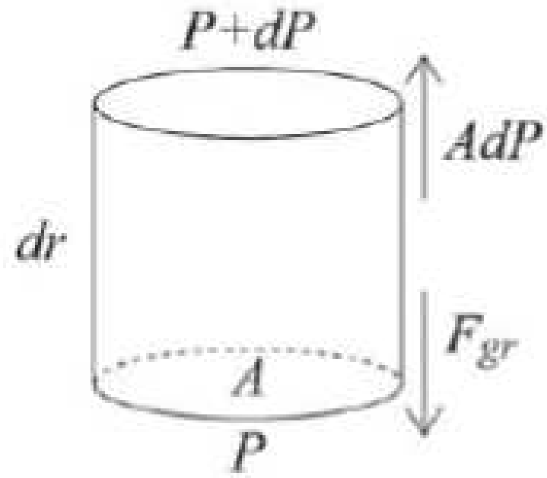
ϵ is given by $\epsilon \equiv (E/A + m_b)\rho$, where E/A is the energy per baryon

Once the energy density and pressure are given, one can find a star with mass M and radius R from pressure-zero condition at the surface.

$$-\frac{GM(r)dm}{r^2} - AdP = 0.$$

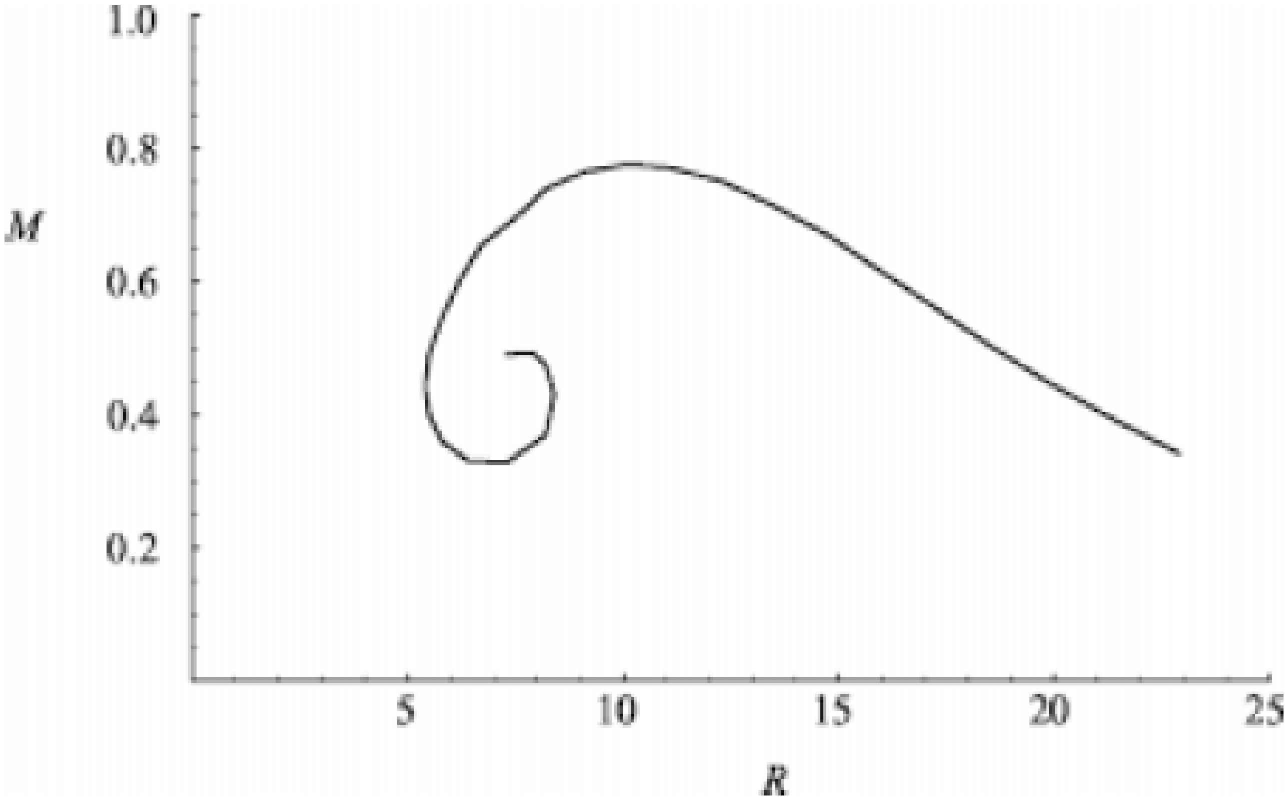
$$dm = \rho(r)Adr$$

$$\boxed{\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}.}$$



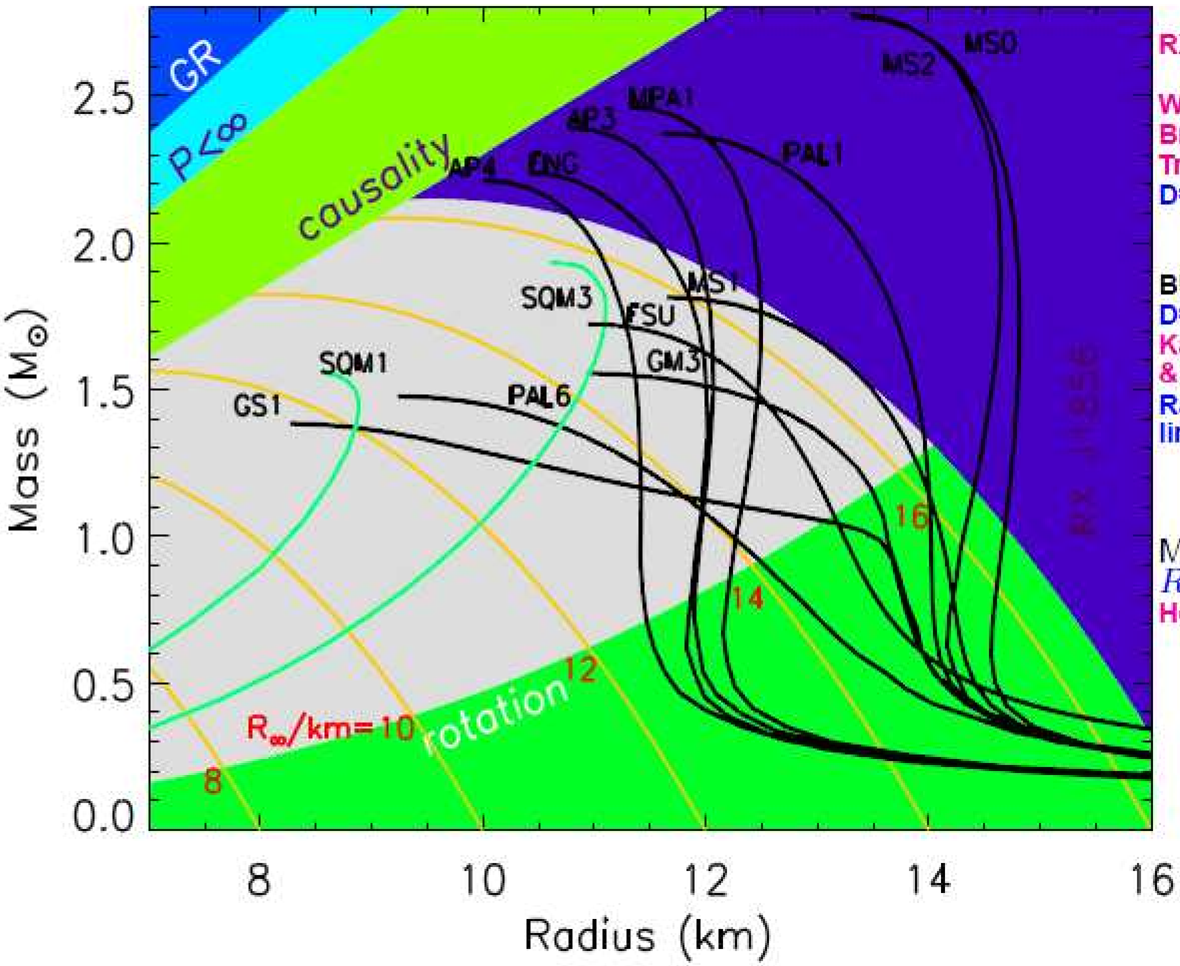
Hydrostatic equilibrium, non-relativistic case

Example: pure neutron stars using a Fermi gas EoS



Richard R. Silbar and Sanjay Reddy

Radiation Radius: Nearby Neutron Star



RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk & Anderson 2002
Raises R_∞ limit to 19.5 km

Magnetic H atmospher
$R_\infty \approx 17$ km
Ho et al. 2007

- An EoS from a dense Sakai–Sugimoto model with a delta function source

K. Y. Kim, S. J. Sin and I. Zahed, JHEP **0801** (2008) 002

The regularized Helmholtz free energy is given by

$$\frac{F_{\rm reg}(n_B)}{V_3} = a\int_{-\infty}^{\infty} dZ K^{2/3}\left(\sqrt{1+\frac{(N_c n_B)^2}{4a^2 b}K^{-5/3}} - 1\right) ,$$

where n_B is the baryon number density and

$$K = 1+Z^2 \ , \ \ a = 3.76\times 10^9 \ {\rm MeV}^4 \ , \ \ b = 7.16\times 10^{-6} \ {\rm MeV}^{-2} \, .$$

Here they took $\lambda \simeq 16.71$ and $M_{\rm KK} \simeq 950$ MeV

cf: M. Rozali, H. H. Shieh, M. Van Raamsdonk and J. Wu, JHEP 0801, 053 (2008);
O. Bergman, G. Lifschytz and M. Lippert, JHEP 0711, 056 (2007).

In this case, however, we could not find a stable compact star, *i.e.*, a star satisfying pressure-zero condition with a radius R, $p(R) = 0$, within a reasonable value of the radius. This means that the EoS from the D4/D8/$\overline{\text{D8}}$ model may not support any stable compact stars or may support one whose radius is very large. This might be due to a deficit of attractive force in the D4/D8/$\overline{\text{D8}}$ since the gravity alone may not balance against the pressure of the D4/D8/$\overline{\text{D8}}$ EoS.

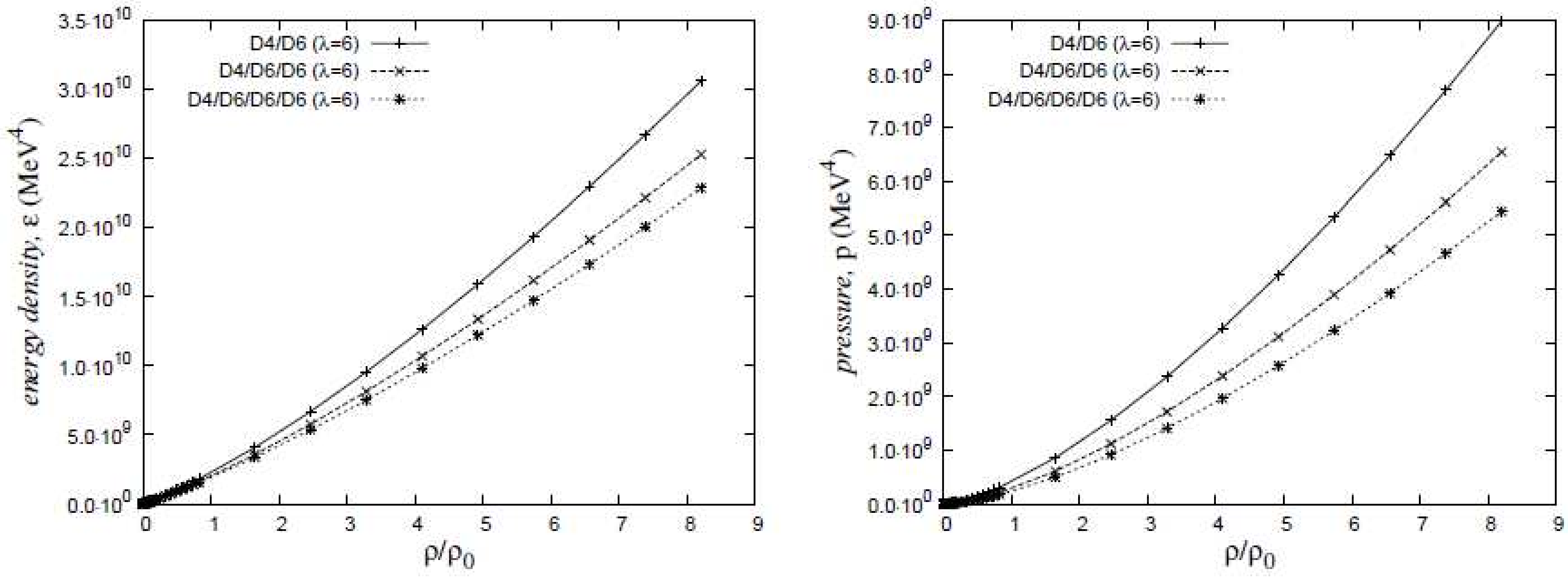
$$\mathcal{L} = \bar{\psi}(i\,\partial\!\!\!/ + g_\sigma\sigma - g_\omega\,\omega\!\!\!/)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega_\mu$$

the pressure of the nuclear matter described by the Walecka is

$$\begin{aligned} P &= \frac{1}{4\pi^2}\left[\frac{2}{3}E_F^*p_F^3 - {m_N^*}^2E_F^*p_F + {m_N^*}^4\ln(\frac{E_F^*+p_F}{m_N^*})\right] \\ &\quad +\frac{1}{2}\frac{g_\omega^2}{m_\omega^2}n^2 - \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}n_s^2\,, \end{aligned}$$

$$\bar{\sigma} = (g_\sigma/m_\sigma^2)n_s \qquad \bar{\omega}_0 = (g_\omega/m_\omega^2)n$$

• EoSs from D4/D6+compact D4 models



(a) (b)

Figure 1: (a): Relation between energy density and baryon number density, and (b): relation between pressure and baryon number density for the three holographic equations of state considered. Here we take $\lambda = 6$ and $M_{\rm KK} = 1.04$ GeV.

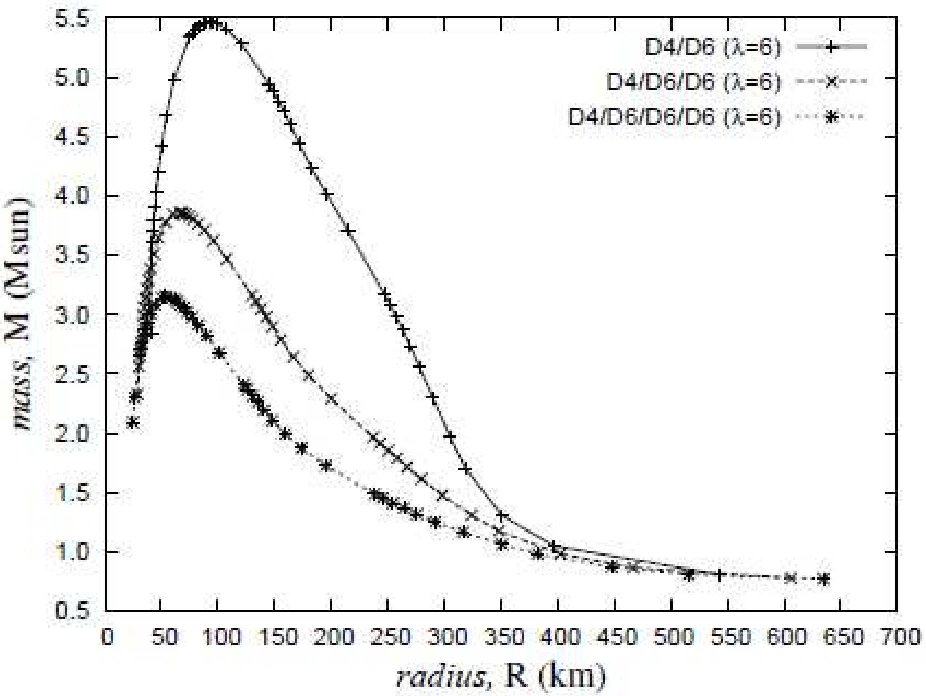
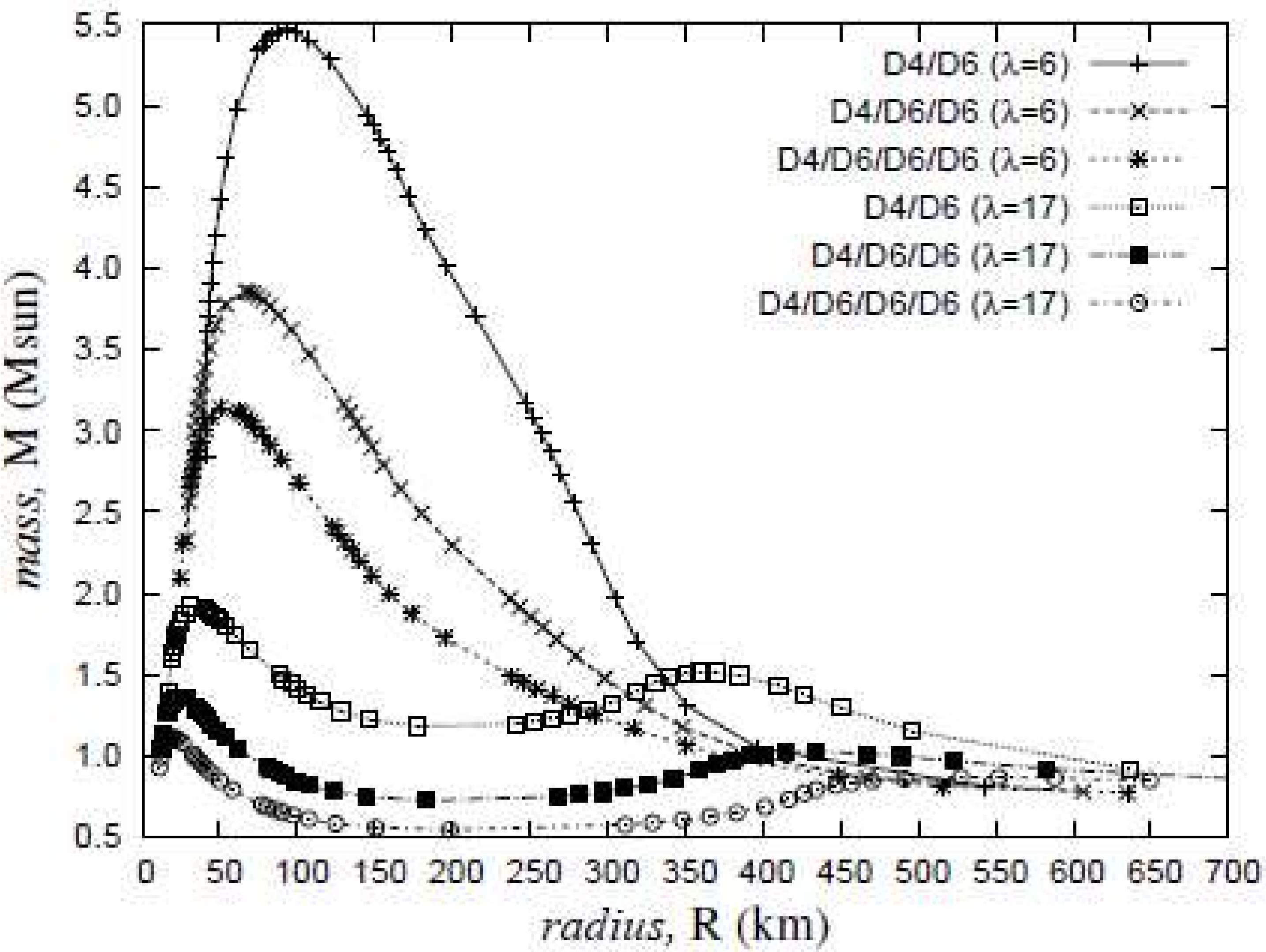
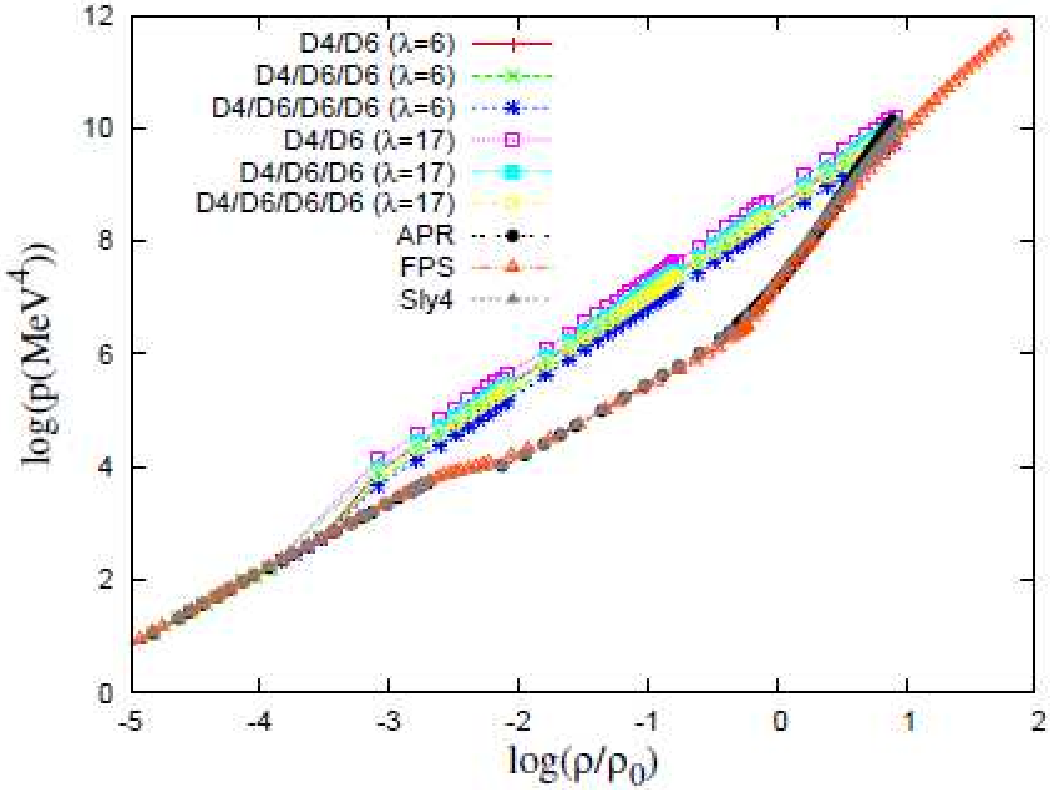


Figure 2: Comparison of mass-radius relations for three holographic equations of state with $\lambda = 6$ and $M_{\mathrm{KK}} = 1.04$ GeV.

For low baryon number densities, i.e., at $\rho < 3 \times 10^{-4} \cdot \rho_0$, the Baym-Pethick-Sutherland equation of state is used



Lack of non-Abelian chiral symmetry? Fermi sea?



Comparison of the pressure from D4/D6 models with those from a few typical EoSs

Summary

- If non-Abelian chiral symmetry is not essential for dense matter, D4/D6 models could be a good tool for dense nuclear matter, especially for isospin physics.
- Having attractive scalar DoF in hQCD models is important.
- Realizing realistic Fermi sea in hQCD is essential towards realistic dense nuclear matter description in hQCD.