Recent progress on anti-kaon–nucleon interactions and dibaryon resonances

Yoichi IKEDA
(RIKEN, Nishina Center)

“Recent progress in hadron physics” --From hadrons to quark and gluon--
@Yonsei Univ., Korea, Feb.19, 2013.
Outline of lectures

✦ Chiral symmetry and chiral dynamics
  ➢ Chiral symmetry in QCD
  ➢ Chiral effective field theory (chiral perturbation theory)
  ➢ Antikaon - nucleon interactions and nature of Λ(1405)

✦ Antikaon in nuclei
  ➢ Antikaon-nucleon potentials model based on chiral dynamics
  ➢ Faddeev equations to handle three-body dynamics (K^barNN - πYN coupled system)
  ➢ Possible spectrum of K^barNN - πYN system

✦ Hadron interactions from lattice QCD
  ➢ Introduction to lattice QCD
  ➢ Hadron scattering on the lattice
  ➢ Application to meson-baryon scattering
(2)
Antikaon in Nuclei
**K^{\text{bar}}\text{-}N and K^{\text{bar}}\text{-}nuclear physics**

✓ **K^{\text{bar}}N (l=0) interaction is...**
- coupled with $\pi\Sigma$ channel
- strongly attractive

⇒ quasi-bound state of $\Lambda(1405)$
- Chiral unitary model $\Rightarrow$ two-pole structure, ($K^{\text{bar}}N$: bound, $\pi\Sigma$: resonance)

Phenomenological construction of $K^{\text{bar}}N$ interaction leads to dense $K^{\text{bar}}$-nuclei

Central density is much larger than normal nuclei

<-- Λ(1405) doorway process to dense matter

✓ Quest for quasi-bound $K^{\text{bar}}\text{-}NN$ systems
- FINUDA, DISTO, J-PARC (E15, E27), ...

✓ $K^{\text{bar}}N$ interaction is a fundamental building block for applications

![Density plots](image-url)
**K\text{bar} -- few-nucleon system**

**Phenomenological parametrization:**
Gaussian form of $K\text{bar}N$ potential (Akaishi-Yamazaki potential)

\[ V_{ij}^{AY}(r) = C_{ij}^{I} \exp[-(r/a_0)^2] \]

\( i, j = K\text{bar}N, \pi\Sigma, \pi\Lambda \)

**Free parameters: coupling constant & range parameter**
1) $K\text{bar}N$ scattering lengths in $I=0, 1$ from partial wave analysis by Martin (’81)
2) $\Lambda(1405)$ binding energy with $K\text{bar}N$ bound state ansatz

\[ U_{K}^{nuc} \]

\( K^{-} + p \)

\( \Sigma + \pi \)

\( \Delta + \pi \)

\( E_{g} = -27 \text{ MeV} \)

\( I_{g} = 40 \text{ MeV} \)

**K\text{bar} -- few-nucleon dynamics: G-matrix:**

G-matrix = T-matrix + Pauli blocking

\[ \bar{K}^{-} + p \]

\[ N \]

\[ g \]

\( \Sigma + \pi \)

\( \Delta + \pi \)

\( -50 \)

\( 0 \)

\( 50 \)

\( 1 \text{ fm} \)

\( 2 \text{ fm} \)

\( 3 \text{ fm} \)

\( \Lambda(1405) \)

Many-body technique applied to $K\text{bar}$-few-nucleon systems
Strange dibaryon as $K^\text{bar}NN$ bound state

Binding energy: 48MeV (below $K^\text{bar}NN$ threshold)
Width: 61MeV

Phenomenological construction of $K^\text{bar}N$ interaction leads to dense $K^\text{bar}$-nuclei

Central density is much larger than normal nuclei
$<-\Lambda(1405)$ doorway process to dense matter

These works motivate recent activities... testing ground: $K^\text{bar}NN$ simplest system
Experimental studies

Stopped $K^-$ reaction on $^6$Li, $^7$Li and $^{12}$C targets (FINUDA exp.)

Enhancement as signal of $K^{\bar{B}NN}$

B.E.$=115$MeV
Width$=67$MeV

Stopped $K^-$ reaction on $^6$Li, $^7$Li and $^{12}$C targets (FINUDA exp.)

Enhancement as signal of $K^{\bar{B}NN}$

B.E.$=115$MeV
Width$=67$MeV

Enhancement as signal of $K^{\bar{B}NN}$

B.E.$=105$MeV
Width$=118$MeV

Enhancement $\sim$100MeV below $K^{\bar{B}NN}$ threshold

--> main component is $K^{\bar{B}NN}$?
"Kaonic nuclear states" in few-body systems

Strange dibaryon as $K^\text{bar}NN$ bound state

Binding energy: 48 MeV (below $K^\text{bar}NN$ threshold)

Width: 61 MeV

Key issue of this investigation is $K^\text{bar}N-\pi\Sigma$ resonance, $\Lambda(1405)$

- It will be very important to take into account full dynamics of $K^\text{bar}N-\pi\Sigma$ system in order to investigate energy of strange dibaryon system

More realistic calculations: coupled-channel

1) Effective $K^\text{bar}N$ potential from bare $K^\text{bar}N-\pi\Sigma$ potential
2) Faddeev equations in coupled-channel formalism

Akaishi, Yamazaki, PRC65 (2002).
Overview

- Binding energy and width of quasi-bound $[K^\text{bar}[NN]_{I=1}]$

**Variational approach**

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**Coupled-channel Faddeev approach**

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**Energy independent**

Overview

**Binding energy and width of quasi-bound \([K^\text{bar}[NN]]_I=1\)**

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**Energy dependent**

1. Akaishi, Yamazaki, PLB535, 70 (2002); PRC76, 045201 (2007).
**Overview**

- **Variational approach**

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- **Coupled-channel Faddeev approach**

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**Topics covered...**

1. Akaishi, Yamazaki, PLB535, 70 (2002); PRC76, 045201 (2007).
Effective $K^\text{bar}N$ potential

- $K^\text{bar}N-\pi Y$ coupled-channel system:
  - Feshbach projection $\rightarrow$ equivalent $K^\text{bar}N$ single-channel effective potential

$$t_{RN-RN} = v_{RN-RN} + v_{RN-RN}G_{0}^{RN}t_{RN-RN} + v_{RN-\pi Y}G_{0}^{\pi Y}t_{\pi Y-RN}$$
$$= v_{eff}^{RN-RN} + v_{eff}^{RN-RN}G_{0}^{RN}t_{RN-RN}$$

$\text{Equivalent } K^\text{bar}N \text{ single-channel effective potential is...}$
- Highly energy dependent due to presence of $\Lambda(1405)$
- Complex (imaginary parts: decay to $\pi Y$ channels)

In chiral SU(3) dynamics, the two-pole structure of \( \Lambda(1405) \) is predicted...

- Different spectral maxima in \( K^{\bar{b}ar}N \) and \( \pi\Sigma \) channels

Chiral effective \( K^{\bar{b}ar}N \) potential must be constructed to reproduce...

- Spectral maximum at 1420 MeV (NOT 1405 MeV)

Effective $K^{\text{bar}}N$ potential (contd.)

- Above $K^{\text{bar}}N$ threshold: good agreement in both models
- $K^{\text{bar}}N$ subthreshold: large deviation
  - Chiral SU(3) dynamics predicts significantly weaker subthreshold attraction than AY potential
  - Solve Schrödinger equation for 3-body $K^{\text{bar}}NN$ system
  - Weak binding system of $K^{\text{bar}}NN (+c.c.)$ system; B.E. ~ 20 MeV

Question:
1) Where is 3-body $\pi\Sigma N$ pole in chiral SU(3) dynamics?
2) If it exists, can the pole affect 3-body spectrum like $\Lambda(1405)$?
   --> explicit dynamics of $\pi\Sigma N$ system is important
   --> Coupled-channel Faddeev approach

Potential model of $K^{\text{bar}}N$ interaction

- **Starting with Tomozawa-Weinberg term**

\[
\mathcal{L}^{\text{TW}} = \frac{i}{8f^2} \text{tr}[\bar{B} \gamma^\mu [\nu_\mu, B]]
\]

\[\nu_\mu = \Phi (\partial_\mu \Phi) - (\partial_\mu \Phi) \Phi + \cdots\]

$\Phi$: PS meson fields, $B$: baryon fields

\[
\text{ derive the potentials from tree level amplitudes }
\]

**Energy-dependent potential (E-dep.)**

\[
V_{ij}(E) = \frac{C_{ij}}{2f^2} (2E - M_i - M_j)
\]

\[C_{ij} = \frac{\lambda_{ij}^{SU(3)}}{16\pi^2 \sqrt{\omega_i \omega_j}}\]

**Energy-independent potential (E-indep.)**

\[
V_{ij} = \frac{C_{ij}}{2f^2} (m_i + m_j)
\]

...energy dependence is fixed at threshold

**Non-perturbative dynamics through Lippmann-Schwinger eq.**

\[
T_{ij}(p_i, p_j; E) = V_{ij}(p_i.p_j) + \sum_n \int dq_n q_n^2 V_{in}(p_i, q_n) \frac{1}{E - E_n(q_n) - \omega_n(q_n) + i\epsilon} T_{nj}(q_n, p_j; E)
\]

**Phenomenological dipole form factor to regularize loop integrals**

\[
V_{ij} \rightarrow g_i(p_i) V_{ij} g_j(p_j)
\]

\[g_i(p_i) = \left(\frac{\Lambda_i^2}{p_i^2 + \Lambda_i^2}\right)^2\]

- Weinberg, PRL 17, 616 (1966).
Cross section & mass spectrum

Cutoff of dipole form factor is determined to reproduce K-p cross section data

**E-dep. potential**

\[ \frac{dN}{dm} \propto |t_{\pi \Sigma - \pi \Sigma}|^2 p_{c.m.} \]

**E-indep. potential**

\[ \frac{dN}{dm} \propto |t_{\pi \Sigma - \pi \Sigma}|^2 p_{c.m.} \]

<table>
<thead>
<tr>
<th></th>
<th>( \Lambda^I_{KN} ) (MeV)</th>
<th>( \Lambda^I_{\pi \Sigma} ) (MeV)</th>
<th>( \Lambda^I_{KN} ) (MeV)</th>
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<th>( \Lambda^I_{\pi \Lambda} ) (MeV)</th>
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<td>E-indep.</td>
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<td>675-725</td>
<td>920</td>
<td>960</td>
<td>640</td>
</tr>
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<td>E-dep.</td>
<td>975-1000</td>
<td>675-725</td>
<td>725</td>
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Pole positions of the $\Lambda(1405)$ [(\Lambda^{\text{KN}}, \Lambda^{\pi\Sigma}) = (1000, 700)\text{MeV}]

$$V_{ij}(E) = \frac{C_{ij}}{2f_{\pi}^2} (2E - M_i - M_j)$$

$$V_{ij} = \frac{C_{ij}}{2f_{\pi}^2} (m_i + m_j)$$

Different subthreshold $K^{\text{bar}}N$ amplitudes would contribute to the energy of strange dibaryon
Three-body equations

Faddeev Equations

\[ T_i(W) = t_i(W, E(p_i)) + \sum_{j \neq i} t_i(W, E(p_i)) G_0(W) T_j(W) \]

\[ t_i(W, E(p_i)) = \nu_i + \nu_i G_0(W) t_i(W, E(p_i)) \]

- **W**: 3-body scattering energy
- **i(j) = 1, 2, 3** (Spectator particles)
- **T(W) = T_1(W) + T_2(W) + T_3(W)** (T: 3-body amplitude)
- **t_i(W, E(p_i))**: 2-body t-matrix with spectator particle i
- **G_0(W)**: 3-body Green’s function
Faddeev equations

\[ T_i(W) = t_i(W, E(p_i)) + \sum_{j \neq i} t_i(W, E(p_i)) G_0(W) T_j(W) \]

\[ t_i(W, E(p_i)) = v_i + v_i G_0(W) t_i(W, E(p_i)) \]

\[ v_i = |g_i\rangle \lambda_i(W, \vec{p}_i) \langle g_i| \]  
(Separable two-body interactions)

Alt-Grassberger-Sandhas (AGS) equations

\[ X_{i,j}(\vec{p}_i, \vec{p}_j; W) = (1 - \delta_{i,j}) Z_{i,j}(\vec{p}_i, \vec{p}_j; W) + \sum_{i \neq n} \int d\vec{p}_n Z_{i,n}(\vec{p}_i, \vec{p}_n; W) \tau_n(\vec{p}_n; W) X_{i,j}(\vec{p}_n, \vec{p}_j; W) \]

\[ X_{ij} = + T_n X_{ij} \]
Coupled-channel AGS equations

\[ X_{i,j}(\bar{p}_i, \bar{p}_j; W) = (1 - \delta_{i,j}) Z_{i,j}(\bar{p}_i, \bar{p}_j; W) \]

\[ + \sum_{i \neq n} \int d\bar{p}_n Z_{i,n}(\bar{p}_i, \bar{p}_n; W) \tau_n(\bar{p}_n; W) X_{i,j}(\bar{p}_n, \bar{p}_j; W) \]

- \[ Z(p_i, p_j; W) : \text{Particle exchange potentials} \]
- \[ \tau(p_n; W) : \text{Isobar propagators} \]

**K^{\text{bar}}_{NN-\pi\Sigma N - \pi\Lambda N} coupled-channel formalism**
Three-body couple-channel analysis is performed by taking into account all s-wave 2-body scatterings in $K^{\text{bar}}NN-\pi YN$ system with $J^P=0^-$. Only the mesonic-decay modes are taken into account.

### Model of pN interactions

- **S11 phase shift**
  - Our scattering length: $0.175m_\pi^{-1}$
  - Exp.: $(0.1788 \pm 0.0050)m_\pi^{-1}$

- **S31 phase shift**
  - Our scattering length: $-0.095m_\pi^{-1}$
  - Exp.: $(-0.0927 \pm 0.0093)m_\pi^{-1}$

### NN potential

$$V_{NN}(q', q) = C_{RR}(q')g_R(q) - C_{AG}(q')g_A(q)$$

<table>
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<th>$\Lambda_A$(MeV)</th>
<th>$C_R$(MeV fm$^3$)</th>
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<td>1215</td>
<td>352</td>
<td>5.05</td>
<td>5.84</td>
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Ikeda, Kamano, Sato, PTP124, 533 (2010).
Results on resonance energies

- Resonance energies are determined by the pole of 3-body amplitudes:
  \[(E, \Gamma) = (2354-2361, 34-46) \text{ & } (2281-2303, 244-320) \text{ MeV} \]
  \[(E, \Gamma) = (2312-2326, 34-40) \text{ MeV} \]
  \[(\text{Energy dependent potential}) \]
  \[(\text{Energy independent potential}) \]

- Two poles in E-dep model are found, but \(\pi \Sigma N\) pole has huge width (250-320 MeV)

\[V_{ij}(E) = \frac{C_{ij}}{2f_{\pi}^2}(2E - M_i - M_j)\]

\[V_{ij} = \frac{C_{ij}}{2f_{\pi}^2}(m_i + m_j)\]

Ikeda, Kamano, Sato, PTP124, 533 (2010).
Signature of strange dibaryon


- Examine possible signatures of strange dibaryon in kaon- and photo-productions (e.g.) E15@J-PARC, LEPS@SPring-8

\[ K^- + {^3He} \rightarrow N + \pi + \Sigma + N \]

- Calculation of “breakup probability” of \((K^\text{bar}N)_{I=0}N\rightarrow\pi\Sigma N\) reaction

\[
\gamma (\pi N)(\pi N) \rightarrow (\pi Y)(\pi Y) + (\pi N)(\pi N) + (\pi N)(\pi N)
\]

\[(Y = \Lambda, \Sigma)\]
Signature of strange dibaryon (contd.)


- Break-up probability of \((K^{\text{bar}}N)_{I=0}N \rightarrow \pi\Sigma N\) reaction

\[
\omega(p'_N, W) = 2\pi \int d^3\vec{p}_N d^3\vec{q}_N \sum \delta \left( W - M - \frac{\vec{p}_N^2}{2\eta_N} - \frac{\vec{q}_N^2}{2\mu_N} \right) \left| T_{\pi\Sigma N - (Y_K)_{I=0}N} (\vec{q}_N, \vec{p}_N, p'_N, W) \right|^2
\]

**E-indep potential:**
- peak around 2310MeV

**E-dep potential:**
- bump around 2350MeV
Signature of strange dibaryon (contd.)

The graph shows the energy distribution of dibaryons in a reaction. The y-axis represents the energy density, and the x-axis represents the mass of the dibaryon. The graph includes two sets of data, one for 'E-dep.' and another for 'E-indep.', with different colors indicating the energy dependence of the particles.

- **E-dep.** data is marked in blue, showing a peak at approximately 15 MeV (shallow) and another at around 55 MeV (deep).
- **E-indep.** data is marked in red, with a peak at about 35 MeV.

The graph also indicates that there is a transition between these energy states, with arrows pointing to the changes in energy levels. The blue circle represents the energy density dependence, while the red circle represents the energy independence.
Summary -- antikaon in nuclei --

**Effective $K^{\text{bar}}N$ potential:**
- Start with coupled-channel
- Feshbach projection onto $K^{\text{bar}}N$ channel --> effective $K^{\text{bar}}N$ potential

**Energy of strange dibaryon:**
- Phenomenological potential / E-indep models --> deep quasi-bound $K^{\text{bar}}NN$ state
- Chiral SU(3) potential models --> shallow quasi-bound $K^{\text{bar}}NN$ state

**Signatures in amplitudes**
- Enhancement observed
- Effect of $\pi\Sigma N$ pole seems quite small due to huge width
- Comparison with experiment:
  initial, (non-mesonic) final state interactions (e.g.) $K^{-}+^{3}\text{He} \rightarrow "K^{\text{bar}}NN"+n \rightarrow \Lambda N+n$