Chiral symmetry breaking - Effective lagrangian

Su Houng Lee[1,*]

[1] *IPAP and Department of Physics, Yonsei University, Seoul 120-749 Korea*

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Abstract

In this lectures, I will introduce an effective Lagrangian with determinant interaction [1]

PACS numbers:

*Electronic mail:suhoung@phya.yonsei.ac.kr

I. A SIMPLE MODEL

There are several ways to introduce the following global invariance to effective model

$$U(2)_L \bigotimes U(2)_R. \tag{1}$$

The quark and meson field transforms as follows

$$\begin{aligned} q'_L &= g_L q_L \\ q'_R &= g_R q_R \\ \Sigma' &= g_L \Sigma g_R^\dagger \end{aligned} \tag{2}$$

A. Linear sigma model

In the linear sigma model, one usually introduces the following representation,

$$\Sigma = \sigma + i\tau\pi \tag{3}$$

Then, we can write an effective Lagrangian involving the fermion and meson fields,

$$\begin{aligned} L &= \bar{q}_L i\partial\!\!\!/ q_L + \bar{q}_R i\partial\!\!\!/ q_R + \frac{1}{4}\mathrm{Tr}\Big(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\Big) \\ &+\frac{1}{4}\mu^2 \mathrm{Tr}\Big(\Sigma\Sigma^\dagger\Big) - \frac{\lambda}{16}\mu^2\Big[\mathrm{Tr}\Big(\Sigma\Sigma^\dagger\Big)\Big]^2 - g\Big(\bar{q}_L \Sigma q_R + \bar{q}_R \Sigma^\dagger q_L\Big) \end{aligned} \tag{4}$$

II. TOY MODEL BY G. 'T HOOFT

Under this transformation, the meson field, which can be thought of being composed of quark-antiquark $\bar{q}_R q_L$, transforms as,

$$\phi' = U^L \phi U_R \tag{5}$$

The meson field are composed of eight meson fields

$$\phi = \frac{1}{2}(\sigma + i\eta) + \frac{1}{2}(\alpha + i\pi)\cdot\tau, \tag{6}$$

Now consider the following Lagrangian

$$\mathcal{L} = -\mathrm{Tr}[\partial_\mu\phi\partial_\mu\phi^\dagger] - V(\phi) \tag{7}$$

A potential V_0 that is invariant under eq.(1) is.

$$\begin{aligned} V_0 &= -\mu^2\mathrm{Tr}[\phi\phi^\dagger] + \frac{1}{2}(\lambda_1 - \lambda_2)\Big(\mathrm{Tr}[\phi\phi^\dagger]\Big)^2 + \frac{1}{2}\lambda_2\mathrm{Tr}[\phi\phi^\dagger]^2 \\ &= -\frac{\mu^2}{2}(\sigma^2+\eta^2+\alpha^2+\pi^2) + \frac{\lambda_1}{8}(\sigma^2+\eta^2+\alpha^2+\pi^2)^2 \\ &\quad +\frac{\lambda_2}{2}\Big((\sigma\alpha+\eta\pi)^2 + (\alpha\times\pi)^2\Big) \end{aligned} \tag{8}$$

Assuming,

$$\langle\sigma\rangle = f, \qquad \sigma = f + s, \tag{9}$$

from

$$\frac{dV(f)}{df^2} = 0 \tag{10}$$

we get

$$f^2 = 2\mu^2/\lambda_1, \tag{11}$$

and,

$$V_0 = \frac{\lambda_1}{2}\Big(fs + \frac{1}{2}(s^2 + \eta^2 + \alpha^2 + \pi^2)\Big)^2 + \frac{\lambda_2}{2}\Big(f\alpha + s\alpha + \eta\pi)^2 + (\alpha \times \pi)^2\Big) \tag{12}$$

from which we read off:

$$m_s^2 = \lambda_1 f^2 = 2\mu^2, \quad m_\eta^2 = 0, \quad m_\alpha^2 = \lambda_2 f^2, \quad m_\pi^2 = 0 \tag{13}$$

There are two possible symmetry breaking terms,

1. Chiral symmetry breaking.

$$\begin{aligned} V_m &= U_m + U_m^*; \\ U_m &= \frac{1}{4} m e^{i\chi} \mathrm{Tr}\phi = \frac{1}{2} m e^{i\chi}(\sigma + i\eta) \end{aligned} \tag{14}$$

2. $U(1)_A$ symmetry breaking.

$$\begin{aligned} V_a &= U_a + U_a^*; \\ U_a &= \kappa e^{i\theta} \det\phi = \kappa e^{i\theta}\Big((\sigma + i\eta)^2 - (\alpha + i\pi)^2\Big) \end{aligned} \tag{15}$$

Note under $U(1)$ transformation

$$\begin{aligned} \phi &\rightarrow e^{i\omega}, \\ (\sigma + i\eta) &\rightarrow e^{i\omega}(\sigma + i\eta), \qquad (\alpha + i\pi) \rightarrow e^{i\omega}(\alpha + i\pi) \end{aligned} \tag{16}$$

A. Explicit chiral symmetry breaking case

In this case,

$$V = V_0 + V_m \tag{17}$$

we can chose $\omega = \pi - \chi$, and

$$V_m = -m\sigma \tag{18}$$

Then

$$f^2 = 2\mu^2/\lambda_1 + 2m/(\lambda_1 f) \tag{19}$$

consequently, we get

$$\begin{aligned} m_s^2 &= 2\mu^2 + 3m/f \\ m_\pi^2 &= m_\eta^2 = m/f \end{aligned} \tag{20}$$

B. $U_A(1)$ symmetry breaking

Consider the other case, where $m = 0$;

$$V = V_0 + V_a \tag{21}$$

we can chose $\omega = \frac{1}{2}(\pi - \theta)$, and

$$V_a = -2\kappa(\sigma^2 + \pi^2 - \eta^2 - \alpha^2) \tag{22}$$

Then

$$f^2 = 2\mu^2/\lambda_1 + 8\kappa/\lambda_1 \tag{23}$$

consequently, we get

$$\begin{aligned} m_\pi^2 &= 0 \\ m_\eta^2 &= 8\kappa \end{aligned} \tag{24}$$

III. VECTOR MESONS

We can explicitly introduce vector mesons by two methods. Gauging external symmetry and using hidden local symmetry.

A. Massive Yang Mills approach

In the massive Yang Mills approach (MYMA), the vector and axial vector mesons are introduced as gauge fields of the external charge. Therefore, the kinetic term in eq. (4) becomes as follows.

$$\begin{aligned} L &= \frac{1}{4} f_\pi^2 \mathrm{Tr}\Big(D_\mu U D^\mu \Sigma^\dagger \Big) \\ &\quad + \frac{1}{2} m_\rho^2 \mathrm{Tr}\Big(A_\mu^L A^{\mu L} + A_\mu^R A^{\mu R} \Big), \end{aligned} \tag{25}$$

here, we reparametrized $\Sigma = f_\pi U$ and used the non-linear realization of the pion fields.

$$D_\mu U = \partial_\mu U - ig A_\mu^L U + ig U A_\mu^R, \tag{26}$$

and $A_\mu^L = V_\mu + A_\mu, A_\mu^R = V_\mu - A_\mu$. This with the vector meson dominance, the external vector field V_μ, will be identified with the vector meson fields by $V_\mu = g_\rho \rho_\mu$. Then one can obtain

$$\begin{aligned} g_{\rho\pi\pi} &= g_\rho \\ m_a^2 &= m_\rho^2 + f_\pi^2 g_\rho^2 \end{aligned} \tag{27}$$

B. Hidden Local Symmetry

The non linear realization that we described above are said to be based on the manifold G/H =SU(2)$_L \bigotimes$SU(2)$_R$/SU(2)$_V$. This is so because the original symmetry $U \rightarrow g_L U g_R^\dagger$ is broken to the vector part only in the vacuum Tr$U \rightarrow$ Tr$gUg^\dagger$. In ref.[2], the hidden local symmetry found by noting that

$$U(x) = \exp[2i\pi(x)/f_\pi] \tag{28}$$

can be rewritten as,

$$U(x) = \xi_L^\dagger \xi_R \tag{29}$$

where the transformation properties for the ξ's are,

$$\begin{aligned} \xi_L(x) &\rightarrow h(x)\xi_L(x)g_L^\dagger, \\ \xi_R(x) &\rightarrow h(x)\xi_R(x)g_R^\dagger \end{aligned} \tag{30}$$

Introducing the gauge field $V(x)_\mu$ for the hidden local symmetry and the covariant derivative, we have the following two lowest order terms,

$$\begin{aligned} L_V &= -\frac{f_\pi^2}{4}\mathrm{Tr}[D_\mu\xi_L\xi_L^\dagger + D_\mu\xi_R\xi_R^\dagger]^2 \\ L_A &= -\frac{f_\pi^2}{4}\mathrm{Tr}[D_\mu\xi_L\xi_L^\dagger - D_\mu\xi_R\xi_R^\dagger]^2 \end{aligned} \tag{31}$$

Then the most general Lagrangian that reproduces the chiral lagrangian in Eq.(4) is given as,

$$L = L_A + aL_V \tag{32}$$

Adding the kinetic term for the vector field we find,

$$\begin{aligned} g_{\rho\pi\pi} &= \frac{1}{2}ag \\ m_\rho^2 &= ag^2 f_\pi^2 \end{aligned} \tag{33}$$

[1] G. 't Hooft, Phys. Rep. **142** 357 (1986).

[2] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett **54** (1985) 1215.