η'

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Abstract

In this lectures, I will discuss the N_c behavior of η' [1].

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I. INTRODUCTION

In pure gauge theory, from the definition of θ vacuum,

$$Z = \int [dA] \exp i \int \mathrm{Tr}\Big[-\frac{1}{4}F^2 + \frac{g^2\theta}{16\pi^2 N} F\tilde{F} \Big], \tag{1}$$

one has the following relation,

$$\begin{aligned} \Big(\frac{d^2E}{d\theta^2}\Big) &= \frac{1}{N^2}\Big(\frac{g^2}{16\pi^2}\Big)^2 \int \langle \mathrm{T}[F\tilde{F}(x), F\tilde{F}(0)]\rangle \\ &= \frac{1}{N^2}\Big(\frac{g^2}{16\pi^2}\Big)^2 \lim_{k\to 0} U(k) \end{aligned} \tag{2}$$

where,

$$U(k) = \int d^4x e^{ikx} \langle \mathrm{T}[F\tilde{F}(x), F\tilde{F}(0)]\rangle \tag{3}$$

It seems there is some puzzle above because of the following,

- The left hand side of Eq. (3) is of order $O(N^2)$ and non vanishing.
- On the other hand, $U(k=0) = 0$ when light quarks are added. However, quark effects are suppressed in $1/N_c$.
- It is believed that infinite sum can do it.

To see how it works, one should note that when light quarks are added,

$$U(k) = \sum_{glueballs} \frac{N^2 a_n^2}{k^2 - M_n^2} + \sum_{mesons} \frac{N c_n^2}{k^2 - m_n^2} = U_0(k) + U_1(k) \tag{4}$$

where,

$$\begin{aligned} N a_n &= \langle 0|F\tilde{F}|n'th \;\; glueball\rangle \\ \sqrt{N} c_n &= \langle 0|F\tilde{F}|n'th \;\; meson\rangle \end{aligned} \tag{5}$$

Note, that except for finite terms, all the mesons mass should have a smooth large N limit $O(1)$. Also, to derive the above counting, one notes that two gluons can be changed to quark-antiquark pair by two coupling g^2; making the counting the same as with quark-aniquark current.

- Therefore, the first terms in $U(k)$ should scale $O(N^2)$ and the second set of terms $O(N)$.
- However, for the sum to be zero at large N limit at $k = 0$, one term in the second sum should have a mass of order $1/N$; this is the η'. That is

$$\frac{N c_{\eta'}^2}{m_{\eta'}^2} = U_0(0) \tag{6}$$

To be more specific, one notes,

1. $$\sqrt{N} c_{\eta'} = \langle 0|F\tilde{F}|\eta'\rangle \tag{7}$$

2. The matrix element is

   $$\partial_\mu J^\mu = \frac{2N_F}{N}\frac{g^2}{16\pi^2}F\tilde{F}, \tag{8}$$

   where J^μ is the U(1) current and N_F the number of light flavors.

3. Therefore,

   $$\langle 0|F\tilde{F}|\eta'\rangle = \frac{16\pi^2}{g^2}\frac{N}{2N_F}\langle 0|\partial_\mu J^\mu|\eta'\rangle \tag{9}$$

4. make use of $f_{\eta'} = f_\pi$ to lowest order in N, then,

   $$\langle 0|\partial_\mu J^\mu|\eta'\rangle = \sqrt{N_F} m_{\eta'}^2 f_\pi \tag{10}$$

5. Therefore, altogether, we have

   $$\frac{N^2}{4N_F} m_{\eta'}^2 f_\pi^2 \Big(\frac{16\pi^2}{g^2}\Big)^2 = U_0(0) \tag{11}$$

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[1] E. Witten, Nucl. Phys. **B 156** 269 (1979).