Gluonic correlations in matter

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We derive the analog of the QCD low-energy theorems for the scalar and pseudoscalar gluonic correlators in nuclear matter. We find that the scalar correlations are depleted while the pseudoscalar correlations are enhanced to leading order in the nuclear matter density. We briefly discuss the consequences of these findings on the QCD spectrum.

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I. INTRODUCTION

QCD low-energy theorems (LET) for the gluonic correlation functions have provided insights into the QCD dynamics both in the vacuum and in the nuclear medium. Indeed, the LET for the pseudoscalar gluon correlation function provides a powerful constraint on the contribution of the η' mass [1,2] and the vacuum gluon condensate [3]. We recall that a quantitative calculation of glueball masses using the QCD sum rule analysis [4,5] requires the use of LET to provide the necessary subtraction constants needed in the dispersion analysis [5,6]. Recently, the extension of the LET to finite temperature and density have been derived for the correlation functions of the scalar gluon operators [7]. These theorems provide strong constraints to be satisfied by lattice calculations or any effective model calculations of QCD in matter. Here, we will derive anew the LET for the scalar gluon correlation functions at low density and show the equivalence of our result to that of Ref. [7]. Then, we will derive similar LET for the pseudoscalar gluonic correlation functions in two ways: First, by assuming that the self-dual gauge configurations are dominant in vacuum and (dilute) matter; second, by using a generic heavy-quark expansion, which gives a model independent result. In both cases, we will derive general relations on the density dependence of the correlation function in terms of differential operators. We briefly comment on the relevance of our results to the QCD spectrum.

II. LET IN VACUUM: REVIEW

Let us start with the definition of the scalar and pseudoscalar gluonic two-point function,

$$S(Q^2)=i\int d^4x e^{iqx}\left\langle T^*\frac{3\alpha_s}{4\pi}G^2(x)\frac{3\alpha_s}{4\pi}G^2(0)\right\rangle,$$

$$P(Q^2)=i\int d^4x e^{iqx}\left\langle T^*\frac{3\alpha_s}{4\pi}G\tilde{G}(x)\frac{3\alpha_s}{4\pi}G\tilde{G}(0)\right\rangle. \quad (1)$$

Here, $G^2=G^a_{\mu\nu}G^a_{\mu\nu}$ and $G\tilde{G}=G^a_{\mu\nu}\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}$. The low-energy theorem for the scalar gluon operator follows from the formula [3]:

$$\frac{d}{d(-1/4g_0^2)}\langle O\rangle=i\int d^4x\langle T^*O(x)g_0^2G^2(0)\rangle, \quad (2)$$

where g_0 is the bare coupling constant of QCD. Now using renormalization-group arguments, one notes that the bare coupling is related to the ultraviolet cutoff M_0 via

$$\langle O\rangle=\text{const}\left[M_0\exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d, \quad (3)$$

with $b=11-\frac{2}{3}N_f$ to one loop. Therefore, the left-hand side of Eq. (2) yields

$$\frac{d}{d(-1/4g_0^2)}\langle O\rangle=d\,\frac{32\pi^2}{b}\langle O\rangle. \quad (4)$$

Substituting $O=(3\alpha_s/4\pi)G^2(0)$ into Eq. (2) and using Eq. (4) we find the LET for the scalar gluon correlation function,

$$S(Q^2=0)=\frac{18}{b}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle. \quad (5)$$

The situation is more subtle for the pseudoscalar gluonic correlation function. Indeed, in the presence of light quarks, the pseudoscalar gluon field is a total differential of the light quark axial current, i.e., $(3\alpha_s/4\pi)G\tilde{G}=\Sigma_q\partial_\mu\overline{q}\gamma_\mu\gamma^5q(m_q\to0)$. Hence,

$$P(Q^2=0)=0. \quad (6)$$

This is not true in the absence of light quarks (NLQ). The approximate LET for this case was originally derived in Ref. [3] by assuming that self-dual field configurations dominate the functional integral [3] over the gauge fields. To show this, consider the QCD functional integral with nonzero θ,

$$Z=\int[dA]\exp(iI), \quad (7)$$

where

$$I=\int d^4x\left[-\frac{1}{4}G^2-\theta\frac{g_0^2}{32\pi^2}G\tilde{G}\right]. \quad (8)$$

Differentiating Eq. (7) twice with respect to θ, we have

$$\frac{\partial^2 \epsilon}{\partial\theta^2} = -i\int d^4x \left\langle T^* \frac{g_0^2}{32\pi^2} G\tilde{G}(x), \frac{g_0^2}{32\pi^2} G\tilde{G}(0) \right\rangle, \quad (9)$$

where ϵ is the vacuum energy density. To evaluate the left-hand side of Eq. (9), we note that the energy density is related to the trace of the energy momentum tensor $\theta_{\mu\nu}$,

$$\epsilon = \frac{1}{4}\langle \theta_\mu^\mu \rangle = \left\langle \frac{\beta(\alpha_s)}{16\alpha_s} G^2 \right\rangle \overset{1-loop}{\approx} \left\langle \frac{-b\alpha_s}{32\pi} G^2 \right\rangle. \quad (10)$$

Now we assume that the self-dual (anti-self-dual) field dominates the functional integral. This changes the Euclidean action as follows:

$$I_E = i\int d^4x \left[-\frac{1}{4g_0^2}\bar{G}^2 - \theta \frac{i}{32\pi^2} \bar{G}\bar{\tilde{G}} \right]$$
$$\to i\int d^4x \left[-\frac{1}{4}\left(\frac{1}{g_0^2} + \frac{i\theta}{8\pi^2} \right) \bar{G}^2 \right], \quad (11)$$

where $\bar{G}$ is obtained by redefining the gauge field $A_\mu \to 1/g_0 \bar{A}_\mu$. Equation (11) implies that within this approximation, the ultraviolet cutoff M_0 is related to θ in a physical quantity by a simple replacement of Eq. (3),

$$\langle O \rangle = \text{const}\left[M_0 \exp\left(-\frac{8\pi^2}{bg_0^2} - \frac{i\theta}{b} \right) \right]^d, \quad (12)$$

for small θ.[1] With Eqs. (12) and (10), it is straightforward to calculate the left-hand side of Eq. (9). We obtain the LET for the pseudoscalar gluonic correlations [3],

$$P_{NLQ}^{sd}(Q^2=0) = -S(Q^2=0) = -\frac{18}{b}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad (13)$$

where the superscript sd refers to the fact that it is based on the self-dual assumption detailed above.

We recall that the difference between the case with light quarks and without light quarks is the contribution due to the η' [1–3], which adds up to give the required zero, i.e.,

$$P(Q^2=0) = P_{NLQ}(Q^2=0) + \frac{\left| \left\langle \frac{3\alpha_s}{4\pi} G\tilde{G} \middle| \eta' \right\rangle \right|^2}{m_{\eta'}^2} = 0. \quad (14)$$

One should note that similar LET for higher point gluonic correlations can be obtained by taking further derivatives either with respect to θ or $-1/4g_0^2$.

[1]We note that for large θ, 2π periodicity is recovered through the branch structure of ϵ.

III. LET FOR THE PSEUDOSCALAR GLUONIC CURRENT: HEAVY QUARK MASS EXPANSION

The above derivation for the pseudoscalar current was obtained within the self-dual approximation in the path integral. This approximation can be averted. Indeed, let us consider a regularization scheme with $\theta \neq 0$ in the presence of a heavy quark with mass m_Q. Substituting the pseudoscalar gluon field for the operator O in Eq. (2), we have

$$\frac{d}{d(-1/4g_0^2)}\left\langle \frac{3\alpha_s}{4\pi} G\tilde{G} \right\rangle = i\int d^4x \left\langle T^* \frac{3\alpha_s}{4\pi} G\tilde{G}(x) g_0^2 G^2(0) \right\rangle. \quad (15)$$

Now, we will make use of the heavy quark mass expansion [8],

$$m_Q \bar{Q} i\gamma_5 Q = -\frac{\alpha_s}{8\pi} G\tilde{G} + O\left(\frac{1}{m_Q^2} \right),$$
$$m_Q \bar{Q} Q = -\frac{\alpha_s}{12\pi} G^2 + O\left(\frac{1}{m_Q^2} \right). \quad (16)$$

Substituting Eq. (16) into Eq. (15) we obtain

$$\frac{d}{d(-1/4g_0^2)}\langle 6 m_Q \bar{Q} i\gamma_5 Q \rangle$$
$$= i\int d^4x \left\langle T^* \frac{3\alpha_s}{4\pi} G\tilde{G}(x) 48\pi^2 m_Q \bar{Q} Q(0) \right\rangle. \quad (17)$$

In general, such substitution is not allowed when the right-hand side of Eq. (17) has an e^{iqx} factor as one is calculating the operator product expansion. However, since we are looking at the LET dominated by a nonperturbative spacelike field configuration [9], such substitution is valid.

Consider now making a chiral rotation in $\theta \to \theta + \pi/2$. This will only affect the heavy quark mass by $m_Q \to m_Q e^{i\gamma_5\pi/2}$. As a result, the scalar quark condensate turns to the pseudoscalar quark condensate and vice versa. Substituting the heavy quark operators back to the gluon operators, we obtain the following:

$$-\frac{2}{32\pi^2} \frac{d}{d(-1/4g_0^2)} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$$
$$= i\int d^4x \left\langle T^* \frac{3\alpha_s}{4\pi} G\tilde{G}(x) \frac{3\alpha_s}{4\pi} G\tilde{G}(0) \right\rangle. \quad (18)$$

Making use of Eq. (4) yields

$$P_{NLQ}^{hq}(Q^2=0) = -\frac{8}{b}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad (19)$$

where the superscript hq refers to the heavy quark mass expansion used. It should be noted, however, that the heavy quark mass can be taken to be infinity so that Eq. (19) is an exact result for QCD with no light quark. This is different from Eq. (13) as obtained within the self-dual approximation. Given the uncertainty in the value of the nonperturbative gluon condensate, both results are consistent with the vacuum phenomenology in Eq. (14).

IV. LET FOR THE SCALAR GLUONIC CURRENT IN NUCLEAR MATTER

Here we derive the LET for the scalar gluonic current at low density. The same derivation holds for finite temperature. The starting point is taking the expectation value of Eq. (2) at finite density. Then we make a low-density expansion of the left-hand side,

$$\frac{d}{d(-1/4g_0^2)}\langle O\rangle_\rho=\frac{d}{d(-1/4g_0^2)}(\langle O\rangle_0+\rho\langle N|O|N\rangle+\mathcal{O}(\rho)). \tag{20}$$

Here, $\langle\cdot\rangle_\rho$ denotes the expectation value at finite density, ρ is the nucleon density, and $|N\rangle$ a nucleon state normalized as $\langle N||N\rangle=(2\pi)^3E_N/m_N\delta^3(0)$. This is the linear density approximation. The derivative of the nucleon expectation value is simply obtained by substituting $d\rightarrow d-3$ in Eq. (4). The derivative of the nucleon density can be obtained from the fact that the chemical potential $\mu=p_f^2/2m_N$ is an external parameter and is independent of g_0. That is, using

$$\frac{d}{d(-1/4g_0^2)}\left(\frac{p_f^2}{2m_N}\right)=0 \tag{21}$$

and

$$\frac{d}{d(-1/4g_0^2)}m_N=\frac{32\pi^2}{b}m_N, \tag{22}$$

we find

$$\frac{d}{d(-1/4g_0^2)}p_f^2=\frac{32\pi^2}{b}2m_N\mu, \tag{23}$$

$$\frac{d}{d(-1/4g_0^2)}\rho=\rho\frac{32\pi^2}{b}\frac{3}{2}. \tag{24}$$

Therefore,

$$\frac{d}{d(-1/4g_0^2)}\langle O\rangle_\rho=\frac{32\pi^2}{b}\left[d\langle O\rangle_0+\left(d-\frac{3}{2}\right)\rho\langle N|O|N\rangle+\mathcal{O}(\rho)\right]. \tag{25}$$

From this, it follows that the changes of the correlator for the scalar gluonic current to leading order in density is

$$\Delta S(Q^2)=S(Q^2;\rho)-S(Q^2;\rho=0)=\frac{45}{4b}\rho\left\langle N\left|\frac{\alpha_s}{\pi}G^2\right|N\right\rangle, \tag{26}$$

where $S(Q^2,\rho)$ is the correlation function defined in Eq. (1) calculated at finite density ρ.

We note that Eq. (25) is also consistent with the result of Ref. [7] applied to leading order in density. In Ref. [7] it was noted that a physical quantity of mass scale d has the following dependence on the temperature or density:

$$\langle O\rangle_{\rho,T}=\Lambda^d f\left(\frac{T}{\Lambda},\frac{\mu}{\Lambda}\right), \tag{27}$$

where $\langle\cdot\rangle_{\rho,T}$ is the expectation value at finite density and temperature, and the mass scale Λ depends on the coupling as in Eq. (3) with $d=1$. Then, it follows that the differential operator acting on the physical parameter can be replaced as

$$\frac{d}{d(-1/4g_0^2)}\langle O\rangle_{\rho,T}=\frac{32\pi^2}{b}\left(d-T\frac{\partial}{\partial T}-\mu\frac{\partial}{\partial\mu}\right)\langle O\rangle_{\rho,T}. \tag{28}$$

When we apply this differential operator to our Eq. (20) and note that $\mu(\partial\rho/\partial\mu)=\frac{3}{2}\rho$, we recover Eq. (25).

V. LET FOR THE PSEUDOSCALAR GLUONIC CURRENT IN NUCLEAR MEDIUM: SELF-DUAL APPROXIMATION

The LET for the pseudoscalar gluonic currents at low density can be similarly obtained in the self-dual approximation. Here, we consider the case with no *sea* light quarks, stressing the effects of matter on the gauge configurations. The starting point is the generalization of Eq. (9) to finite density. The difference compared to the scalar case is in taking the derivative with respect to θ instead of g_0^2. Again, using the linear density approximation, we have

$$\frac{d^2}{d\theta^2}\langle O\rangle_\rho=\frac{d^2}{d\theta^2}[\langle O\rangle_0+\rho\langle N|O|N\rangle+\mathcal{O}(\rho)]. \tag{29}$$

To evaluate the derivatives with respect to θ, we only need the relation in Eq. (12) and the fact that $d\mu/d\theta=0$. To leading order in density, we have

$$\frac{d\rho}{d\theta}=\frac{3}{2}\left(-\frac{i}{b}\right)\rho,$$

$$\frac{d^2\rho}{d\theta^2}=\frac{9}{4}\left(-\frac{i}{b}\right)^2\rho. \tag{30}$$

Therefore, we have

$$\frac{d^2}{d\theta^2}\langle O\rangle_\rho=\left(\frac{-i}{b}\right)^2\left[d^2\langle O\rangle_0+\left(\frac{9}{4}+3(d-3)+(d-3)^2\right)\rho\langle N|O|N\rangle+\mathcal{O}(\rho)\right]. \quad (31)$$

Substituting this into Eq. (9) gives

$$\Delta P^{sd}_{NLQ}(Q^2=0)=P^{sd}_{NLQ}(Q^2=0;\rho)-P^{sd}_{NLQ}(Q^2=0;0)=-\frac{225}{32b}\rho\left\langle N\left|\frac{\alpha_s}{\pi}G^2\right|N\right\rangle. \quad (32)$$

We can also derive a general formula for Eq. (31) in terms of differential operators involving the derivatives with respect to the temperature and/or chemical potential. Again, we have the θ dependence of a physical parameter through Eq. (27) and also Eq. (12). Therefore, we have

$$\frac{d^n}{d\theta^n}\langle O\rangle_{\rho,T}=\left(\frac{-i}{b}\right)^n\left(d-T\frac{\partial}{\partial T}-\mu\frac{\partial}{\partial\mu}\right)^n\langle O\rangle_{\rho,T}, \quad (33)$$

which is consistent with Eq. (31). In the linear density approximation Eq. (33) can be written as

$$\frac{d^n}{d\theta^n}(\langle O\rangle_0+\rho\langle N|O|N\rangle)=\left(\frac{-i}{b}\right)^n\left[d^n\langle O\rangle_0+\left(d-\frac{3}{2}\right)^n\rho\langle N|O|N\rangle\right]. \quad (34)$$

VI. LET FOR THE PSEUDOSCALAR GLUONIC CURRENT IN NUCLEAR MEDIUM: HEAVY QUARK EXPANSION

The generalization of the low-energy theorem for the pseudoscalar current based on the heavy quark expansion can be obtained in the same way as in the scalar case. The derivation provides for a check on the self-duality assumption. Starting from the expectation value of Eq. (18) at finite density, and substituting Eq. (25) into the left-hand side, we obtain

$$\Delta P^{hq}_{NLQ}(Q^2=0)=P^{hq}_{NLQ}(Q^2=0;\rho)-P^{hq}_{NLQ}(Q^2=0;0)=-\frac{5}{b}\rho\left\langle N\left|\frac{\alpha_s}{\pi}G^2\right|N\right\rangle. \quad (35)$$

This is an exact result for QCD with no light quark. Note that the overall factor is different from Eq. (32), but the sign is the same. The relation in terms of differential operators can be obtained in the same way as in the scalar case by substituting Eq. (28) into the left-hand side of Eq. (18).

VII. SUMMARY

We have derived the LET for scalar and pseudoscalar gluonic correlation functions at low density. Although, we have restricted our discussions to two-point functions, we can easily generalize this method to higher-point functions and to finite temperature. Since Eq. (14) is still valid in the medium, we expect from Eq. (35),

$$\Delta\left(\frac{\left|\left\langle\frac{3\alpha_s}{4\pi}G\tilde{G}\middle|\eta'\right\rangle\right|^2}{m^2_{\eta'}}\right)=-\Delta P_{NLQ}(Q^2=0)<0. \quad (36)$$

Equation (36) is a model independent constraint to leading order in density that any model calculation of QCD at finite density has to satisfy. This is especially useful considering the persisting difficulties of lattice calculations at finite density. When combined with a leading density behavior of the residue at finite density, Eq. (36) can be used to obtain the change in the η' mass to leading order in density. The fate of the η' mass in matter may have interesting consequences on low mass dilepton emissions, pion thresholds, as well as the strong CP problem in matter.

Note added in proof. After submission, we found that our result in Eq. (19) was consistent with Ref. [10].

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