# Large $N_c$

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## Abstract

In this lectures, I will discuss the basics of large $N_c$ Physics given in [1].

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## I. INTRODUCTION

QCD has no explicit expansion parameter. Therefore, the hypothetical limit of large $N_c$ could give us some insight. It served as the basis for using tree graph physics of effective theories. We are going to show the following in the $N_c \to \infty$ limit.

1. Mesons and glue states are free, stable, non-interacting; decay amplitude $1/\sqrt{N_c}$ and meson-meson elastic scattering amplitudes $1/N_c$. Given by tree graphs of infinite sum of meson exchanges of mesons.

2. Zweig's rule is exact at $N_c \to \infty$. Mesons are pure $q\bar{q}$ states (no $\bar{q}qqq$ states).

To prove this, the basic starting point is the following

1. $q_i$, $\bar{q}^j$ indices have $i, j = N_c$ possible color lines with opposite arrow.

2. $A_i^j$ scales line $q_i$, $\bar{q}^j$

3. The scaling of gluon line and gluon self energy scales the same. $\to$ This implies the QCD coupling scales as $g \to 1/\sqrt{N_c}$.

## II. FEYNMAN DIAGRAMS FOR LARGE $N_c$

The consequences for Feynman diagrams are as follows.

1. Planar diagrams dominate. Non-Planar diagram are suppressed as $1/N_c^2$.

2. Internal quark lines are suppressed as $1/N_c$.

For two point function of quark bilinears,

1. Again, planar diagrams dominate.

2. Internal quark loops are suppressed.

3. External gluon lines are suppressed. Only quarks are at the edge.

## III. PROPERTIES OF MESONS FOR LARGE $N_c$

Here we combine arguments of confinement with large $N_c$. Let the two point function be

$$\langle J(k)J(-k)\rangle = \int d^4x e^{ikx} \bar{q}\Gamma q(x), \bar{q}\Gamma q(0)\rangle \tag{1}$$

1. From previous argument, one notes that this two point function scales as $N_c$, with quark lines at the edge and planar diagrams only. Now, by cutting the planar diagrams, one notes that the only color singlet combination is where the quark antiquark line and all the intermediate gluon lines are included. This is just like the color index flow in the following representation of the intermediate state.

$$q^i A_i^j A_j^k A_k^l q_l \tag{2}$$

Since we have confinement, the states can only be a single meson. Hence a spectral representation will be possible.

$$\langle J(k)J(-k)\rangle = \sum_n \frac{a_n^2}{k^2 - m_n^2} \tag{3}$$

2. The sum in Eq. (3) should be infinity. This is so because the asymptotic behavior ($k \to \infty$) of the left hand side is of $\log k^2$. To obtained similar behavior in the right hand side, the sum has to go to infinity.
3. The masses should not have any imaginary part in the same limit. Therefore the usual meson mass should have a smooth large $N_c$ limit.
4. $a_n \propto \sqrt{N_c}$.
5. Looking at three point function $\langle JJJ\rangle$, one notes three meson coupling scales as $1/\sqrt{N_c}$. Therefore, the decay rate of meson is also of the same order and dominated by the two body decay.
6. Looking at the four point function, one notes that elastic scattering is of order $1/N_c$.

### A. Attractiveness of the large $N_c$ limit

1. Since quark loops are suppressed, mesons are pure $\bar{q}q$ state with no $\bar{q}q$ sea. No exotic.
2. Mesons are usually nonet rather than octet and singlet. The only difference between singlet and octet is contributions from disconnected diagrams, which are suppressed at large $N_c$.
3. Multibody decay proceed via resonant two-body states. Many examples.
4. Regge phenomena.

## IV. BARYONS IN THE LARGE $N_c$ LIMIT: NON-RELATIVISTIC LIMIT

Diagram itself depends on $N_c$, because a baryon has $N_c$ antisymmetric quarks.

1. (Gluon exchange $1/N_c$ from two coupling) $\times$ (${}_NC_2 = \frac{N(N-1)}{2} \propto N_c^2$) $= N_c$
2. But mass itself is of order $N_C$. Hatree approximation

$$\begin{aligned} H &= Mass + H_{KE} + H_{int} \\ &= NM + \sum\Big(-\frac{\partial_i^2}{2M}\Big) - \frac{g^2}{N}\sum_{i<j}\frac{1}{|x_i - x_j|} \\ &= N\Big(M + T + \frac{1}{2}V\Big) \end{aligned} \tag{4}$$

## V. SCATTERING PROCESS

1. Baryon-Baryon interaction: (from the 1st baryon $N_c$ ) $\times$ ( from the 2nd baryon $N_c$) $\times$ (interaction $g^2 = 1/N_c$) = order $N_c$ and same as Baryon mass.

2. Baryon-meson interaction: (from 1st baryon $N_c$ ) $\times$ ( from the 2nd meson $O(1)$) $\times$ (interaction $g^2 = 1/N_c$) = order 1 and suppressed compared to the Baryon mass.

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[1] E. Witten, Nucl. Phys. **B 160** 57 (1980).